

Exponents (Level 2)**Question Paper, Set 03**

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the value of $\sqrt[3]{5^6}$. [1 mark]
2. If $a^3 = 5^6$ and $b^5 = 2^{10}$, find ab . [2 marks]
3. Simplify $125^2 \times 5^{-5}$. [1 mark]
4. Find the value of $\frac{(81)^{\frac{3}{4}} \times (8)^{\frac{2}{3}}}{(36)^{\frac{1}{2}}}$. [2 marks]
5. Find the value of $\left(\frac{8}{125}\right)^{-\frac{1}{3}} + \left(\frac{1}{2}\right)^{-2} - 3 \times 16^{\frac{1}{4}} \times 7^0$. [3 marks]
6. Simplify $\frac{8^{\frac{3}{5}} \times 5^{\frac{1}{4}} \times 27^{\frac{2}{3}}}{3^{\frac{9}{5}} \times 10^{\frac{1}{4}}} \div \frac{4^{\frac{3}{5}} \times 8^{\frac{2}{5}}}{2^{\frac{1}{4}} \times 12^{\frac{1}{5}}}$. [5 marks]
7. Show that $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1$. [3 marks]
8. If $a + b + c = 0$, show that $x^{-\frac{a^2}{bc}} \cdot x^{-\frac{b^2}{ca}} \cdot x^{-\frac{c^2}{ab}} = \frac{1}{x^3}$. [5 marks]
9. Solve $5^{x+2} = 625$. [1 mark]
10. Solve $3^{2x-5} \times 7^{x-2} = 21$. [2 marks]
11. If $5^x = 15$, find 5^{x+2} . [1 mark]
12. Solve $\left(\frac{2}{5}\right)^x \left(\frac{5}{2}\right)^{3x} = \frac{625}{16}$. [2 marks]
13. Solve $8^{x-1} = 2^{3x-2} - 8$. [5 marks]
14. If $\sqrt{3^n} = 729$, find $2^{\frac{n}{4}}$. [3 marks]
15. If $2^p = 3$, $3^q = 5$, $5^r = 7$ and $7^t = 16$, find $pqrt$. [2 marks]
16. If $3^x = 5^y = 45^{-z}$, show that $\frac{2}{x} + \frac{1}{y} + \frac{1}{z} = 0$. [5 marks]
17. Find the value of $(0.0081)^{\frac{1}{4}}$. [1 mark]
18. If $x = 2^{\frac{2}{3}} + 2^{\frac{4}{3}}$, show that $x^3 - 12x = 20$. [3 marks]
19. If $\frac{x^2}{x^{0.5}} = 27$ and $x > 0$, find x . [2 marks]

20. Find the value of $\left\{ (5^{-3})^2 \right\}^{-\frac{1}{3}}$.

[1 mark]

Key sheet

Q	Answer
1	25
2	$ab = 100$
3	5
4	18
5	$\frac{1}{2}$
6	6
7	Shown: $\text{LHS} = 1 = \text{RHS}$
8	Shown: $\text{LHS} = \frac{1}{x^3} = \text{RHS}$
9	$x = 2$
10	$x = 3$
11	375
12	$x = 2$
13	$x = 2$
14	8
15	$pqrt = 4$
16	Shown: $\frac{2}{x} + \frac{1}{y} + \frac{1}{z} = 0$
17	0.3
18	Shown: $x^3 - 12x = 20$
19	$x = 9$
20	25

Solutions

1. Find the value of $\sqrt[3]{5^6}$. [1]

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\sqrt[3]{5^6} = 5^{\frac{6}{3}} = 5^2 = 25$$

Ans: 25

2. If $a^3 = 5^6$ and $b^5 = 2^{10}$, find ab . [2]

$$a^m = b^n \Rightarrow a = b^{\frac{n}{m}}$$

$$a = 5^{\frac{6}{3}} = 5^2 = 25$$

$$b = 2^{\frac{10}{5}} = 2^2 = 4$$

$$ab = 25 \times 4 = 100$$

Ans: $ab = 100$

3. Simplify $125^2 \times 5^{-5}$. [1]

$$= (5^3)^2 \times 5^{-5}$$

$$= 5^6 \times 5^{-5}$$

$$= 5^{6-5} = 5^1$$

$$= 5$$

Ans: 5

4. Find the value of $\frac{(81)^{\frac{3}{4}} \times (8)^{\frac{2}{3}}}{(36)^{\frac{1}{2}}}$. [2]

$$= \frac{(3^4)^{\frac{3}{4}} \times (2^3)^{\frac{2}{3}}}{(6^2)^{\frac{1}{2}}}$$

$$= \frac{3^3 \times 2^2}{6}$$

$$= \frac{27 \times 4}{6}$$

$$= 18$$

Ans: 18

5. Find the value of $\left(\frac{8}{125}\right)^{-\frac{1}{3}} + \left(\frac{1}{2}\right)^{-2} - 3 \times 16^{\frac{1}{4}} \times 7^0$. [3]

$$= \left[\left(\frac{2}{5}\right)^3\right]^{-\frac{1}{3}} + 2^2 - 3 \times (2^4)^{\frac{1}{4}} \times 1$$

$$= \frac{5}{2} + 4 - 3 \times 2$$

$$= \frac{5}{2} + 4 - 6$$

$$= \frac{5+8-12}{2}$$

$$= \frac{1}{2}$$

Ans: $\frac{1}{2}$

6. Simplify $\frac{8^{\frac{3}{5}} \times 5^{\frac{1}{4}} \times 27^{\frac{2}{3}}}{3^{\frac{9}{5}} \times 10^{\frac{1}{4}}} \div \frac{4^{\frac{3}{5}} \times 8^{\frac{2}{5}}}{2^{\frac{1}{4}} \times 12^{\frac{2}{5}}}$. [5]

$$\begin{aligned}
 &= \frac{8^{\frac{3}{5}} \times 5^{\frac{1}{4}} \times 27^{\frac{2}{3}}}{3^{\frac{9}{5}} \times 10^{\frac{1}{4}}} \times \frac{2^{\frac{1}{4}} \times 12^{\frac{2}{5}}}{4^{\frac{3}{5}} \times 8^{\frac{2}{5}}} \\
 &= \frac{(2^3)^{\frac{3}{5}} \times 5^{\frac{1}{4}} \times (3^3)^{\frac{2}{3}}}{3^{\frac{9}{5}} \times (2 \times 5)^{\frac{1}{4}}} \times \frac{2^{\frac{1}{4}} \times (2^2 \times 3)^{\frac{2}{5}}}{(2^2)^{\frac{3}{5}} \times (2^3)^{\frac{2}{5}}} \\
 &= \frac{2^{\frac{9}{5}} \times 5^{\frac{1}{4}} \times 3^2}{3^{\frac{9}{5}} \times 2^{\frac{1}{4}} \times 5^{\frac{1}{4}}} \times \frac{2^{\frac{1}{4}} \times 2^{\frac{4}{5}} \times 3^{\frac{4}{5}}}{2^{\frac{6}{5}} \times 2^{\frac{4}{5}}} \\
 &= 2^{\frac{9}{5} - \frac{1}{4} - \frac{6}{5} - \frac{4}{5} + \frac{1}{4} + \frac{8}{5}} \times 3^{2 - \frac{9}{5} + \frac{4}{5}} \times 5^{\frac{1}{4} - \frac{1}{4}} \\
 &= 2^1 \times 3^1 \times 5^0 \\
 &= 6
 \end{aligned}$$

Ans: 6

7. Show that $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1$. [3]

$$\begin{aligned}
 \text{LHS} &= x^{\frac{a-b}{ab}} \cdot x^{\frac{b-c}{bc}} \cdot x^{\frac{c-a}{ca}} \\
 \frac{a-b}{ab} &= \frac{1}{b} - \frac{1}{a}, \quad \frac{b-c}{bc} = \frac{1}{c} - \frac{1}{b}, \quad \frac{c-a}{ca} = \frac{1}{a} - \frac{1}{c} \\
 &= x^{\frac{1}{b} - \frac{1}{a} + \frac{1}{c} - \frac{1}{b} + \frac{1}{a} - \frac{1}{c}} \\
 &= x^0 = 1 = \text{RHS}
 \end{aligned}$$

Ans: Shown: LHS = 1 = RHS

8. If $a + b + c = 0$, show that $x^{-\frac{a^2}{bc}} \cdot x^{-\frac{b^2}{ca}} \cdot x^{-\frac{c^2}{ab}} = \frac{1}{x^3}$. [5]

Given $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$

$$\begin{aligned}
 \text{LHS} &= x^{-\left(\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right)} \\
 &= x^{-\frac{a^3 + b^3 + c^3}{abc}} \\
 &= x^{-\frac{3abc}{abc}} = x^{-3} = \frac{1}{x^3} = \text{RHS}
 \end{aligned}$$

Ans: Shown: LHS = $\frac{1}{x^3}$ = RHS

9. Solve $5^{x+2} = 625$. [1]

$$\begin{aligned}
 5^{x+2} &= 5^4 \\
 \Rightarrow x + 2 &= 4 \\
 \Rightarrow x &= 2
 \end{aligned}$$

Ans: $x = 2$

10. Solve $3^{2x-5} \times 7^{x-2} = 21$. [2]

$$\begin{aligned}
 21 &= 3^1 \times 7^1 \\
 \Rightarrow 3^{2x-5} \times 7^{x-2} &= 3^1 \times 7^1 \\
 \Rightarrow 2x - 5 &= 1 \text{ and } x - 2 = 1 \\
 \Rightarrow x &= 3 \text{ and } x = 3 \\
 \therefore x &= 3
 \end{aligned}$$

Ans: $x = 3$

11. If $5^x = 15$, find 5^{x+2} . [1]

$$\begin{aligned}
 5^{x+2} &= 5^x \times 5^2 \\
 &= 15 \times 25 = 375
 \end{aligned}$$

Ans: 375

12. Solve $\left(\frac{2}{5}\right)^x \left(\frac{5}{2}\right)^{3x} = \frac{625}{16}$. [2]

$$\begin{aligned}\left(\frac{2}{5}\right)^x &= \left(\frac{5}{2}\right)^{-x} \\ \Rightarrow \left(\frac{5}{2}\right)^{-x} \cdot \left(\frac{5}{2}\right)^{3x} &= \left(\frac{5}{2}\right)^4 \\ \Rightarrow \left(\frac{5}{2}\right)^{2x} &= \left(\frac{5}{2}\right)^4 \\ \Rightarrow 2x &= 4 \Rightarrow x = 2\end{aligned}$$

Ans: $x = 2$

13. Solve $8^{x-1} = 2^{3x-2} - 8$. [5]

$$\begin{aligned}(2^3)^{x-1} &= 2^{3x-2} - 8 \\ \Rightarrow 2^{3x-3} &= 2^{3x-2} - 8 \\ \Rightarrow \frac{2^{3x}}{8} &= \frac{2^{3x}}{4} - 8 \\ \Rightarrow 2^{3x} \left(\frac{1}{4} - \frac{1}{8}\right) &= 8 \\ \Rightarrow 2^{3x} \times \frac{1}{8} &= 8 \\ \Rightarrow 2^{3x} &= 8 \times 8 = 64 = 2^6 \\ \Rightarrow 3x &= 6 \Rightarrow x = 2\end{aligned}$$

Ans: $x = 2$

14. If $\sqrt{3^n} = 729$, find $2^{\frac{n}{4}}$. [3]

$$\begin{aligned}3^{\frac{n}{2}} &= 729 = 3^6 \\ \Rightarrow \frac{n}{2} &= 6 \Rightarrow n = 12 \\ 2^{\frac{n}{4}} &= 2^{\frac{12}{4}} = 2^3 = 8\end{aligned}$$

Ans: 8

15. If $2^p = 3$, $3^q = 5$, $5^r = 7$ and $7^t = 16$, find $pqrt$. [2]

$$\begin{aligned}5 &= 3^q = (2^p)^q = 2^{pq} \\ 7 &= 5^r = (2^{pq})^r = 2^{pqr} \\ 16 &= 7^t = (2^{pqr})^t = 2^{pqrt} \\ 2^{pqrt} &= 16 = 2^4 \\ \Rightarrow pqrt &= 4\end{aligned}$$

Ans: $pqrt = 4$

16. If $3^x = 5^y = 45^{-z}$, show that $\frac{2}{x} + \frac{1}{y} + \frac{1}{z} = 0$. [5]

$$\begin{aligned}\text{Let } 3^x = 5^y = 45^{-z} &= k \text{ (say)} \\ \Rightarrow 3 &= k^{\frac{1}{x}}, 5 = k^{\frac{1}{y}}, 45 = k^{-\frac{1}{z}} \\ 45 &= 3^2 \times 5 \\ \Rightarrow k^{-\frac{1}{z}} &= \left(k^{\frac{1}{x}}\right)^2 \times k^{\frac{1}{y}} \\ \Rightarrow k^{-\frac{1}{z}} &= k^{\frac{2}{x} + \frac{1}{y}} \\ \Rightarrow -\frac{1}{z} &= \frac{2}{x} + \frac{1}{y} \\ \Rightarrow \frac{2}{x} + \frac{1}{y} + \frac{1}{z} &= 0\end{aligned}$$

Ans: Shown: $\frac{2}{x} + \frac{1}{y} + \frac{1}{z} = 0$

17. Find the value of $(0.0081)^{\frac{1}{4}}$. [1]

$$0.0081 = \frac{81}{10000} = \left(\frac{3}{10}\right)^4 = (0.3)^4$$

$$(0.0081)^{\frac{1}{4}} = [(0.3)^4]^{\frac{1}{4}} = 0.3$$

Ans: 0.3

18. If $x = 2^{\frac{2}{3}} + 2^{\frac{4}{3}}$, show that $x^3 - 12x = 20$. [3]

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$x^3 = \left(2^{\frac{2}{3}}\right)^3 + \left(2^{\frac{4}{3}}\right)^3 + 3 \cdot 2^{\frac{2}{3}} \cdot 2^{\frac{4}{3}} \left(2^{\frac{2}{3}} + 2^{\frac{4}{3}}\right)$$

$$= 4 + 16 + 3 \times 4 \times x$$

$$x^3 = 20 + 12x$$

$$\Rightarrow x^3 - 12x = 20$$

Ans: Shown: $x^3 - 12x = 20$

19. If $\frac{x^2}{x^{0.5}} = 27$ and $x > 0$, find x . [2]

$$0.5 = \frac{1}{2}$$

$$x^{2-\frac{1}{2}} = 27 \Rightarrow x^{\frac{3}{2}} = 27$$

$$x = 27^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^2 = 9$$

Ans: $x = 9$

20. Find the value of $\left\{(5^{-3})^2\right\}^{-\frac{1}{3}}$. [1]

$$= 5^{(-3) \times 2 \times (-\frac{1}{3})}$$

$$= 5^2$$

$$= 25$$

Ans: 25
