

Exponents (Level 2)**Question Paper, Set 02**

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $a^3 = 2^{15}$, find a . [1 mark]
2. If $a^3 = 4^6$ and $b^2 = 3^6$ ($b > 0$), find $\frac{a}{b}$. [2 marks]
3. Simplify $16^3 \times 2^{-10}$. [1 mark]
4. Find the value of $\frac{(16)^{\frac{3}{4}} \times (125)^{\frac{2}{3}}}{(100)^{\frac{1}{2}}}$. [2 marks]
5. Find the value of $\left(\frac{1}{3}\right)^{-3} - 2 \times 27^{\frac{2}{3}} \times 5^0 + \left(\frac{25}{4}\right)^{-\frac{1}{2}}$. [3 marks]
6. Simplify $\frac{5^{\frac{6}{5}} \times 9^{\frac{6}{5}} \times 2^{\frac{4}{5}}}{15 \times 8^{\frac{1}{5}}} \div \frac{15^{\frac{1}{5}} \times 9^{\frac{1}{5}}}{12^{\frac{2}{5}} \times 3^{\frac{4}{5}}}$. [5 marks]
7. Show that $\left(\frac{x^a}{x^b}\right)^{a+b-c} \cdot \left(\frac{x^b}{x^c}\right)^{b+c-a} \cdot \left(\frac{x^c}{x^a}\right)^{c+a-b} = 1$. [3 marks]
8. If $a + b + c = 0$, show that $x^{\frac{2a^2}{bc}} \cdot x^{\frac{2b^2}{ca}} \cdot x^{\frac{2c^2}{ab}} = x^6$. [5 marks]
9. Solve $10^x = 0.0001$. [1 mark]
10. Solve $5^{x-1} \times 2^{3x-4} = 800$. [2 marks]
11. If $2^x = 192$, find 2^{x-6} . [1 mark]
12. Solve $\left(\sqrt{\frac{2}{3}}\right)^{x-1} = \frac{27}{8}$. [2 marks]
13. Solve $4^{x+1} = 2^{2x+3} - 64$. [5 marks]
14. If $4^{x+3} = 8^{x+1}$, find 3^{x-1} . [3 marks]
15. If $3^x = 5$, $5^y = 7$ and $7^z = 9$, find xyz . [2 marks]
16. If $2^x = 3^y = 72^z$, show that $\frac{1}{z} = \frac{3}{x} + \frac{2}{y}$. [5 marks]
17. Find the value of $\sqrt[3]{27^{-2}}$. [1 mark]
18. If $x = 5^{\frac{1}{3}} + 25^{\frac{1}{3}}$, show that $x^3 - 15x = 30$. [3 marks]
19. Find the value of $(625)^{0.17} \times (625)^{0.08}$. [2 marks]

20. Find the value of $\left\{(2^4)^{-3}\right\}^{-\frac{1}{4}}$.

[1 mark]

Key sheet

Q	Answer
1	$a = 32$
2	$\frac{16}{27}$
3	4
4	20
5	$\frac{47}{5}$
6	18
7	Shown: $\text{LHS} = 1 = \text{RHS}$
8	Shown: $\text{LHS} = x^6 = \text{RHS}$
9	$x = -4$
10	$x = 3$
11	3
12	$x = -5$
13	$x = 2$
14	9
15	$xyz = 2$
16	Shown: $\frac{1}{z} = \frac{3}{x} + \frac{2}{y}$
17	$\frac{1}{9}$
18	Shown: $x^3 - 15x = 30$
19	5
20	8

Solutions

1. If $a^3 = 2^{15}$, find a . [1]

$$a^m = b^n \Rightarrow a = b^{\frac{n}{m}}$$

$$a = 2^{\frac{15}{3}} = 2^5 = 32$$

Ans: $a = 32$

2. If $a^3 = 4^6$ and $b^2 = 3^6$ ($b > 0$), find $\frac{a}{b}$. [2]

$$a^m = b^n \Rightarrow a = b^{\frac{n}{m}}$$

$$a = 4^{\frac{6}{3}} = 4^2 = 16$$

$$b = 3^{\frac{6}{2}} = 3^3 = 27$$

$$\frac{a}{b} = \frac{16}{27}$$

Ans: $\frac{16}{27}$

3. Simplify $16^3 \times 2^{-10}$. [1]

$$= (2^4)^3 \times 2^{-10}$$

$$= 2^{12} \times 2^{-10}$$

$$= 2^{12-10} = 2^2$$

$$= 4$$

Ans: 4

4. Find the value of $\frac{(16)^{\frac{3}{4}} \times (125)^{\frac{2}{3}}}{(100)^{\frac{1}{2}}}$. [2]

$$= \frac{(2^4)^{\frac{3}{4}} \times (5^3)^{\frac{2}{3}}}{(10^2)^{\frac{1}{2}}}$$

$$= \frac{2^3 \times 5^2}{10}$$

$$= \frac{8 \times 25}{10}$$

$$= 20$$

Ans: 20

5. Find the value of $\left(\frac{1}{3}\right)^{-3} - 2 \times 27^{\frac{2}{3}} \times 5^0 + \left(\frac{25}{4}\right)^{-\frac{1}{2}}$. [3]

$$= 3^3 - 2 \times (3^3)^{\frac{2}{3}} \times 1 + \left[\left(\frac{5}{2}\right)^2\right]^{-\frac{1}{2}}$$

$$= 27 - 2 \times 3^2 + \frac{2}{5}$$

$$= 27 - 18 + \frac{2}{5}$$

$$= \frac{135-90+2}{5}$$

$$= \frac{47}{5}$$

Ans: $\frac{47}{5}$

6. Simplify $\frac{5^{\frac{6}{5}} \times 9^{\frac{6}{5}} \times 2^{\frac{4}{5}}}{15 \times 8^{\frac{1}{5}}} \div \frac{15^{\frac{1}{5}} \times 9^{\frac{1}{5}}}{12^{\frac{2}{5}} \times 3^{\frac{4}{5}}}$. [5]

$$\begin{aligned}
 &= \frac{5^{\frac{6}{5}} \times 9^{\frac{6}{5}} \times 2^{\frac{4}{5}}}{15 \times 8^{\frac{1}{5}}} \times \frac{12^{\frac{2}{5}} \times 3^{\frac{4}{5}}}{15^{\frac{1}{5}} \times 9^{\frac{1}{5}}} \\
 &= \frac{5^{\frac{6}{5}} \times (3^2)^{\frac{6}{5}} \times 2^{\frac{4}{5}}}{(3 \times 5) \times (2^3)^{\frac{1}{5}}} \times \frac{(2^2 \times 3)^{\frac{2}{5}} \times 3^{\frac{4}{5}}}{(3 \times 5)^{\frac{1}{5}} \times (3^2)^{\frac{1}{5}}} \\
 &= \frac{5^{\frac{6}{5}} \times 3^{\frac{12}{5}} \times 2^{\frac{4}{5}}}{3 \times 5 \times 2^{\frac{3}{5}}} \times \frac{2^{\frac{4}{5}} \times 3^{\frac{2}{5}} \times 3^{\frac{4}{5}}}{3^{\frac{1}{5}} \times 5^{\frac{1}{5}} \times 3^{\frac{2}{5}}} \\
 &= 2^{\frac{4}{5} - \frac{3}{5} + \frac{4}{5}} \times 3^{\frac{12}{5} - 1 - \frac{1}{5} - \frac{2}{5} + \frac{2}{5} + \frac{4}{5}} \times 5^{\frac{6}{5} - 1 - \frac{1}{5}} \\
 &= 2^1 \times 3^2 \times 5^0 \\
 &= 18
 \end{aligned}$$

Ans: 18

7. Show that $\left(\frac{x^a}{x^b}\right)^{a+b-c} \cdot \left(\frac{x^b}{x^c}\right)^{b+c-a} \cdot \left(\frac{x^c}{x^a}\right)^{c+a-b} = 1$. [3]

$$\begin{aligned}
 \text{LHS} &= x^{(a-b)(a+b-c)} \cdot x^{(b-c)(b+c-a)} \cdot x^{(c-a)(c+a-b)} \\
 (a-b)(a+b-c) &= a^2 - b^2 - ca + bc \\
 (b-c)(b+c-a) &= b^2 - c^2 - ab + ca \\
 (c-a)(c+a-b) &= c^2 - a^2 - bc + ab \\
 \text{Adding these, every term cancels, so the power is 0} \\
 \text{LHS} &= x^0 = 1 = \text{RHS}
 \end{aligned}$$

Ans: Shown: LHS = 1 = RHS

8. If $a + b + c = 0$, show that $x^{\frac{2a^2}{bc}} \cdot x^{\frac{2b^2}{ca}} \cdot x^{\frac{2c^2}{ab}} = x^6$. [5]

$$\begin{aligned}
 \text{Given } a + b + c &= 0 \Rightarrow a^3 + b^3 + c^3 = 3abc \\
 \text{LHS} &= x^{\frac{2a^2}{bc} + \frac{2b^2}{ca} + \frac{2c^2}{ab}} \\
 &= x^{\frac{2(a^3 + b^3 + c^3)}{abc}} \\
 &= x^{\frac{2 \times 3abc}{abc}} = x^6 = \text{RHS}
 \end{aligned}$$

Ans: Shown: LHS = x^6 = RHS

9. Solve $10^x = 0.0001$. [1]

$$\begin{aligned}
 10^x &= 0.0001 = \frac{1}{10^4} = 10^{-4} \\
 \Rightarrow x &= -4 \\
 \Rightarrow x &= -4
 \end{aligned}$$

Ans: $x = -4$

10. Solve $5^{x-1} \times 2^{3x-4} = 800$. [2]

$$\begin{aligned}
 800 &= 5^2 \times 2^5 \\
 \Rightarrow 5^{x-1} \times 2^{3x-4} &= 5^2 \times 2^5 \\
 \Rightarrow x-1 &= 2 \text{ and } 3x-4 = 5 \\
 \Rightarrow x &= 3 \text{ and } x = 3 \\
 \therefore x &= 3
 \end{aligned}$$

Ans: $x = 3$

11. If $2^x = 192$, find 2^{x-6} . [1]

$$\begin{aligned} 2^{x-6} &= \frac{2^x}{2^6} \\ &= \frac{192}{64} = 3 \end{aligned}$$

Ans: 3

12. Solve $\left(\sqrt{\frac{2}{3}}\right)^{x-1} = \frac{27}{8}$. [2]

$$\begin{aligned} \left[\left(\frac{2}{3}\right)^{\frac{1}{2}}\right]^{x-1} &= \left(\frac{3}{2}\right)^3 \\ \Rightarrow \left(\frac{2}{3}\right)^{\frac{x-1}{2}} &= \left(\frac{2}{3}\right)^{-3} \\ \Rightarrow \frac{x-1}{2} &= -3 \\ \Rightarrow x-1 &= -6 \\ \Rightarrow x &= -5 \end{aligned}$$

Ans: $x = -5$

13. Solve $4^{x+1} = 2^{2x+3} - 64$. [5]

$$\begin{aligned} (2^2)^{x+1} &= 2^{2x+3} - 64 \\ \Rightarrow 2^{2x+2} &= 2^{2x+3} - 64 \\ \Rightarrow 4 \times 2^{2x} &= 8 \times 2^{2x} - 64 \\ \Rightarrow 2^{2x} (8 - 4) &= 64 \\ \Rightarrow 2^{2x} \times 4 &= 64 \\ \Rightarrow 2^{2x} &= 64 \times \frac{1}{4} = 16 = 2^4 \\ \Rightarrow 2x &= 4 \Rightarrow x = 2 \end{aligned}$$

Ans: $x = 2$

14. If $4^{x+3} = 8^{x+1}$, find 3^{x-1} . [3]

$$\begin{aligned} (2^2)^{x+3} &= (2^3)^{x+1} \\ \Rightarrow 2^{2x+6} &= 2^{3x+3} \\ \Rightarrow 2x+6 &= 3x+3 \Rightarrow x = 3 \\ 3^{x-1} &= 3^{3-1} = 3^2 = 9 \end{aligned}$$

Ans: 9

15. If $3^x = 5$, $5^y = 7$ and $7^z = 9$, find xyz . [2]

$$\begin{aligned} 7 &= 5^y = (3^x)^y = 3^{xy} \\ 9 &= 7^z = (3^{xy})^z = 3^{xyz} \\ 3^{xyz} &= 9 = 3^2 \\ \Rightarrow xyz &= 2 \end{aligned}$$

Ans: $xyz = 2$

16. If $2^x = 3^y = 72^z$, show that $\frac{1}{z} = \frac{3}{x} + \frac{2}{y}$. [5]

Let $2^x = 3^y = 72^z = k$ (say)

$$\Rightarrow 2 = k^{\frac{1}{x}}, 3 = k^{\frac{1}{y}}, 72 = k^{\frac{1}{z}}$$

$$72 = 2^3 \times 3^2$$

$$\Rightarrow k^{\frac{1}{z}} = \left(k^{\frac{1}{x}}\right)^3 \times \left(k^{\frac{1}{y}}\right)^2$$

$$\Rightarrow k^{\frac{1}{z}} = k^{\frac{3}{x} + \frac{2}{y}}$$

$$\Rightarrow \frac{1}{z} = \frac{3}{x} + \frac{2}{y}$$

Ans: Shown: $\frac{1}{z} = \frac{3}{x} + \frac{2}{y}$

17. Find the value of $\sqrt[3]{27^{-2}}$. [1]

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\sqrt[3]{27^{-2}} = 27^{-\frac{2}{3}} = (3^3)^{-\frac{2}{3}}$$

$$= 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

Ans: $\frac{1}{9}$

18. If $x = 5^{\frac{1}{3}} + 25^{\frac{1}{3}}$, show that $x^3 - 15x = 30$. [3]

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$x^3 = \left(5^{\frac{1}{3}}\right)^3 + \left(25^{\frac{1}{3}}\right)^3 + 3 \cdot 5^{\frac{1}{3}} \cdot 25^{\frac{1}{3}} \left(5^{\frac{1}{3}} + 25^{\frac{1}{3}}\right)$$

$$= 5 + 25 + 3 \times 5 \times x$$

$$x^3 = 30 + 15x$$

$$\Rightarrow x^3 - 15x = 30$$

Ans: Shown: $x^3 - 15x = 30$

19. Find the value of $(625)^{0.17} \times (625)^{0.08}$. [2]

$$= (625)^{0.17+0.08} = (625)^{0.25}$$

$$0.25 = \frac{1}{4}$$

$$= (5^4)^{\frac{1}{4}} = 5$$

Ans: 5

20. Find the value of $\left\{(2^4)^{-3}\right\}^{-\frac{1}{4}}$. [1]

$$= 2^{4 \times (-3) \times (-\frac{1}{4})}$$

$$= 2^3$$

$$= 8$$

Ans: 8
