

Exponents (Level 2)**Question Paper, Set 10**

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the value of $\left(-\frac{1}{3}\right)^{-3}$. [1 mark]
2. Find the value of $\sqrt[5]{4^{10}} \times \sqrt[3]{6^{-3}}$. [2 marks]
3. Simplify $343^2 \times 49^{-3}$. [1 mark]
4. Find the value of $\frac{(128)^{\frac{3}{7}} \times (81)^{\frac{1}{4}}}{(144)^{\frac{1}{2}}}$. [2 marks]
5. Find the value of $\left(\frac{25}{36}\right)^{-\frac{1}{2}} + \left(\frac{8}{27}\right)^{-\frac{2}{3}} - \frac{\sqrt{100}}{\sqrt[3]{125}}$. [3 marks]
6. Simplify $\frac{5^{\frac{8}{5}} \times 8^{\frac{1}{4}} \times 3^{\frac{3}{5}}}{6^{-\frac{4}{5}} \times 15^{\frac{2}{5}}} \div \frac{12^{\frac{4}{5}} \times 2^{\frac{3}{4}}}{27^{\frac{1}{5}} \times 12^{\frac{2}{5}}}$. [5 marks]
7. Simplify $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c}$. [3 marks]
8. If $p + q + r = 0$, show that $2^{p^3} \cdot 2^{q^3} \cdot 2^{r^3} = 8^{pqr}$. [5 marks]
9. Solve $5^{4x} = 0.04$. [1 mark]
10. Solve $7^{x-1} \times 2^{2x-1} = 56$. [2 marks]
11. If $10^{3y} = 216$, find 10^{-y} . [1 mark]
12. Solve $27^{2x} = \frac{1}{81}$. [2 marks]
13. Solve $4^{x+1} - 2^{2x+1} = 128$. [5 marks]
14. If $125^{x-1} = 25$, find 27^x . [3 marks]
15. If $10^x = 3$, $3^y = 5$ and $5^z = 0.001$, find xyz . [2 marks]
16. If $2^x = 5^y = 1000^z$, show that $z = \frac{xy}{3(x+y)}$. [5 marks]
17. Find the value of $\sqrt[6]{64^{-5}}$. [1 mark]
18. Find the value of $(2^{\frac{1}{3}} + 3^{\frac{1}{3}})(2^{\frac{2}{3}} - 6^{\frac{1}{3}} + 3^{\frac{2}{3}})$. [3 marks]
19. If $\frac{x^{0.5}}{x^2} = \frac{1}{8}$ and $x > 0$, find x . [2 marks]

20. Find the value of $\left\{(10^{-3})^2\right\}^{-\frac{1}{2}}$.

[1 mark]

Key sheet

Q	Answer
1	-27
2	$\frac{8}{3}$
3	1
4	2
5	$\frac{29}{20}$
6	15
7	$x^{a^2-c^2}$
8	Shown: both sides $= 2^{3pqr}$
9	$x = -\frac{1}{2}$
10	$x = 2$
11	$\frac{1}{6}$
12	$x = -\frac{2}{3}$
13	$x = 3$
14	243
15	$xyz = -3$
16	Shown: $z = \frac{xy}{3(x+y)}$
17	$\frac{1}{32}$
18	5
19	$x = 4$
20	1000

Solutions

1. Find the value of $(-\frac{1}{3})^{-3}$. [1]

$$\begin{aligned}\left(\frac{a}{b}\right)^{-n} &= \left(\frac{b}{a}\right)^n \\ \left(-\frac{1}{3}\right)^{-3} &= (-3)^3 \\ &= -27 \text{ (3 is odd)}\end{aligned}$$

Ans: -27

2. Find the value of $\sqrt[5]{4^{10}} \times \sqrt[3]{6^{-3}}$. [2]

$$\begin{aligned}\sqrt[5]{4^{10}} &= 4^{\frac{10}{5}} = 4^2 = 16 \\ \sqrt[3]{6^{-3}} &= 6^{\frac{-3}{3}} = 6^{-1} = \frac{1}{6} \\ 16 \times \frac{1}{6} &= \frac{8}{3}\end{aligned}$$

Ans: $\frac{8}{3}$

3. Simplify $343^2 \times 49^{-3}$. [1]

$$\begin{aligned}&= (7^3)^2 \times (7^2)^{-3} \\ &= 7^6 \times 7^{-6} \\ &= 7^{6-6} = 7^0 \\ &= 1\end{aligned}$$

Ans: 1

4. Find the value of $\frac{(128)^{\frac{3}{7}} \times (81)^{\frac{1}{4}}}{(144)^{\frac{1}{2}}}$. [2]

$$\begin{aligned}&= \frac{(2^7)^{\frac{3}{7}} \times (3^4)^{\frac{1}{4}}}{(12^2)^{\frac{1}{2}}} \\ &= \frac{2^3 \times 3}{12} \\ &= \frac{8 \times 3}{12} \\ &= 2\end{aligned}$$

Ans: 2

5. Find the value of $(\frac{25}{36})^{-\frac{1}{2}} + (\frac{8}{27})^{-\frac{2}{3}} - \frac{\sqrt{100}}{\sqrt[3]{125}}$. [3]

$$\begin{aligned}&= \left[\left(\frac{5}{6}\right)^2\right]^{-\frac{1}{2}} + \left[\left(\frac{2}{3}\right)^3\right]^{-\frac{2}{3}} - \frac{\sqrt{10^2}}{\sqrt[3]{5^3}} \\ &= \frac{6}{5} + \left(\frac{3}{2}\right)^2 - \frac{10}{5} \\ &= \frac{6}{5} + \frac{9}{4} - 2 \\ &= \frac{24+45-40}{20} \\ &= \frac{29}{20}\end{aligned}$$

Ans: $\frac{29}{20}$

6. Simplify $\frac{5^{\frac{8}{5}} \times 8^{\frac{1}{4}} \times 3^{\frac{3}{5}}}{6^{-\frac{4}{5}} \times 15^{\frac{3}{5}}} \div \frac{12^{\frac{4}{5}} \times 2^{\frac{3}{4}}}{27^{\frac{1}{5}} \times 12^{\frac{2}{5}}}$. [5]

$$\begin{aligned}
 &= \frac{5^{\frac{8}{5}} \times 8^{\frac{1}{4}} \times 3^{\frac{3}{5}}}{6^{-\frac{4}{5}} \times 15^{\frac{3}{5}}} \times \frac{27^{\frac{1}{5}} \times 12^{\frac{2}{5}}}{12^{\frac{4}{5}} \times 2^{\frac{3}{4}}} \\
 &= \frac{5^{\frac{8}{5}} \times (2^3)^{\frac{1}{4}} \times 3^{\frac{3}{5}}}{(2 \times 3)^{-\frac{4}{5}} \times (3 \times 5)^{\frac{3}{5}}} \times \frac{(3^3)^{\frac{1}{5}} \times (2^2 \times 3)^{\frac{2}{5}}}{(2^2 \times 3)^{\frac{4}{5}} \times 2^{\frac{3}{4}}} \\
 &= \frac{5^{\frac{8}{5}} \times 2^{\frac{3}{4}} \times 3^{\frac{3}{5}}}{2^{-\frac{4}{5}} \times 3^{-\frac{4}{5}} \times 3^{\frac{3}{5}} \times 5^{\frac{3}{5}}} \times \frac{3^{\frac{3}{5}} \times 2^{\frac{4}{5}} \times 3^{\frac{2}{5}}}{2^{\frac{8}{5}} \times 3^{\frac{4}{5}} \times 2^{\frac{3}{4}}} \\
 &= 2^{\frac{3}{4} + \frac{4}{5} - \frac{8}{5} - \frac{3}{4} + \frac{4}{5}} \times 3^{\frac{3}{5} + \frac{4}{5} - \frac{3}{5} - \frac{4}{5} + \frac{3}{5} + \frac{2}{5}} \times 5^{\frac{8}{5} - \frac{3}{5}} \\
 &= 2^0 \times 3^1 \times 5^1 \\
 &= 15
 \end{aligned}$$

Ans: 15

7. Simplify $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c}$. [3]

$$\begin{aligned}
 &= (x^{a-b})^{a+b} \cdot (x^{b-c})^{b+c} \\
 &= x^{a^2-b^2} \cdot x^{b^2-c^2} \\
 &= x^{a^2-b^2+b^2-c^2} \\
 &= x^{a^2-c^2}
 \end{aligned}$$

Ans: $x^{a^2-c^2}$

8. If $p + q + r = 0$, show that $2^{p^3} \cdot 2^{q^3} \cdot 2^{r^3} = 8^{pqr}$. [5]

Given $p + q + r = 0 \Rightarrow p^3 + q^3 + r^3 = 3pqr$

LHS = $2^{p^3+q^3+r^3} = 2^{3pqr}$

= $(2^3)^{pqr} = 8^{pqr} = \text{RHS}$

Ans: Shown: both sides = 2^{3pqr}

9. Solve $5^{4x} = 0.04$. [1]

$5^{4x} = 0.04 = \frac{1}{5^2} = 5^{-2}$

$\Rightarrow 4x = -2$

$\Rightarrow x = -\frac{1}{2}$

Ans: $x = -\frac{1}{2}$

10. Solve $7^{x-1} \times 2^{2x-1} = 56$. [2]

$56 = 7^1 \times 2^3$

$\Rightarrow 7^{x-1} \times 2^{2x-1} = 7^1 \times 2^3$

$\Rightarrow x - 1 = 1 \text{ and } 2x - 1 = 3$

$\Rightarrow x = 2 \text{ and } x = 2$

$\therefore x = 2$

Ans: $x = 2$

11. If $10^{3y} = 216$, find 10^{-y} . [1]

$(10^y)^3 = 6^3$

$\Rightarrow 10^y = 6$

$10^{-y} = \frac{1}{10^y} = \frac{1}{6}$

Ans: $\frac{1}{6}$

12. Solve $27^{2x} = \frac{1}{81}$. [2]

$$\begin{aligned}(3^3)^{2x} &= \frac{1}{3^4} \\ \Rightarrow 3^{6x} &= 3^{-4} \\ \Rightarrow 6x &= -4 \\ \Rightarrow x &= -\frac{2}{3} \\ \text{Ans: } x &= -\frac{2}{3}\end{aligned}$$

13. Solve $4^{x+1} - 2^{2x+1} = 128$. [5]

$$\begin{aligned}(2^2)^{x+1} - 2^{2x+1} &= 128 \\ \Rightarrow 2^{2x+2} - 2^{2x+1} &= 128 \\ \Rightarrow 4 \times 2^{2x} - 2 \times 2^{2x} &= 128 \\ \Rightarrow 2^{2x} (4 - 2) &= 128 \\ \Rightarrow 2^{2x} \times 2 &= 128 \\ \Rightarrow 2^{2x} &= 128 \times \frac{1}{2} = 64 = 2^6 \\ \Rightarrow 2x &= 6 \Rightarrow x = 3 \\ \text{Ans: } x &= 3\end{aligned}$$

14. If $125^{x-1} = 25$, find 27^x . [3]

$$\begin{aligned}(5^3)^{x-1} &= 5^2 \\ \Rightarrow 5^{3x-3} &= 5^2 \Rightarrow 3x - 3 = 2 \Rightarrow x = \frac{5}{3} \\ 27^x &= (3^3)^{\frac{5}{3}} = 3^5 = 243 \\ \text{Ans: } 243\end{aligned}$$

15. If $10^x = 3$, $3^y = 5$ and $5^z = 0.001$, find xyz . [2]

$$\begin{aligned}5 &= 3^y = (10^x)^y = 10^{xy} \\ 0.001 &= 5^z = (10^{xy})^z = 10^{xyz} \\ 10^{xyz} &= 0.001 = 10^{-3} \\ \Rightarrow xyz &= -3 \\ \text{Ans: } xyz &= -3\end{aligned}$$

16. If $2^x = 5^y = 1000^z$, show that $z = \frac{xy}{3(x+y)}$. [5]

$$\begin{aligned}\text{Let } 2^x &= 5^y = 1000^z = k \text{ (say)} \\ \Rightarrow 2 &= k^{\frac{1}{x}}, 5 = k^{\frac{1}{y}}, 1000 = k^{\frac{1}{z}} \\ 1000 &= 2^3 \times 5^3 \\ \Rightarrow k^{\frac{1}{z}} &= \left(k^{\frac{1}{x}}\right)^3 \times \left(k^{\frac{1}{y}}\right)^3 \\ \Rightarrow k^{\frac{1}{z}} &= k^{\frac{3}{x} + \frac{3}{y}} \\ \Rightarrow \frac{1}{z} &= \frac{3}{x} + \frac{3}{y} \\ \Rightarrow \frac{1}{z} &= \frac{3y+3x}{xy} = \frac{3(x+y)}{xy} \\ \Rightarrow z &= \frac{xy}{3(x+y)} \\ \text{Ans: Shown: } z &= \frac{xy}{3(x+y)}\end{aligned}$$

17. Find the value of $\sqrt[6]{64^{-5}}$. [1]

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\sqrt[6]{64^{-5}} = 64^{-\frac{5}{6}} = (2^6)^{-\frac{5}{6}}$$

$$= 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$$

Ans: $\frac{1}{32}$

18. Find the value of $(2^{\frac{1}{3}} + 3^{\frac{1}{3}})(2^{\frac{2}{3}} - 6^{\frac{1}{3}} + 3^{\frac{2}{3}})$. [3]

$$(a + b)(a^2 + b^2 - ab) = a^3 + b^3 \text{ with } a = 2^{\frac{1}{3}}, b = 3^{\frac{1}{3}}$$

$$ab = 2^{\frac{1}{3}} \cdot 3^{\frac{1}{3}} = 6^{\frac{1}{3}}$$

$$\text{Value} = (2^{\frac{1}{3}})^3 + (3^{\frac{1}{3}})^3 = 2 + 3 = 5$$

Ans: 5

19. If $\frac{x^{0.5}}{x^2} = \frac{1}{8}$ and $x > 0$, find x . [2]

$$x^{\frac{1}{2}-2} = \frac{1}{8} \Rightarrow x^{-\frac{3}{2}} = 2^{-3}$$

$$\Rightarrow x^{\frac{3}{2}} = 2^3 = 8$$

$$x = 8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^2 = 4$$

Ans: $x = 4$

20. Find the value of $\left\{ (10^{-3})^2 \right\}^{-\frac{1}{2}}$. [1]

$$= 10^{(-3) \times 2 \times (-\frac{1}{2})}$$

$$= 10^3$$

$$= 1000$$

Ans: 1000
