EXPRESSWALA ACADEMY

Exponents (Level 2)

Question Paper, Set 01

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Write $\sqrt[5]{7^4}$ in exponent form. [1 mark]

2. Simplify $\left(\frac{2}{3}\right)^{-3} \times \left(\frac{3}{4}\right)^{-2}$. [2 marks]

3. Simplify $27^2 \times 3^{-5}$. [1 mark]

4. Find the value of $\frac{(36)^{\frac{3}{2}} \times (32)^{\frac{2}{5}}}{(27)^{\frac{2}{3}}}$. [2 marks]

5. Find the value of $\left(\frac{27}{64}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{16}{81}\right)^{\frac{1}{4}}} + \frac{\sqrt{25}}{\sqrt[3]{27}}$. [3 marks]

6. Simplify $\frac{6^{\frac{1}{5}} \times 9^{-\frac{1}{5}} \times 12}{3^{\frac{8}{5}} \times 15^{-\frac{4}{5}}} \div \frac{5^{\frac{6}{5}} \times 2^{\frac{7}{5}}}{8^{\frac{2}{5}} \times 25^{\frac{1}{5}}}$. [5 marks]

7. Show that $\left(\frac{x^a}{x^b}\right)^c \cdot \left(\frac{x^b}{x^c}\right)^a \cdot \left(\frac{x^c}{x^a}\right)^b = 1$. [3 marks]

8. If $a + b + c = 0$, show that $x^{\frac{a^2}{2bc}} \cdot x^{\frac{b^2}{2ca}} \cdot x^{\frac{c^2}{2ab}} = x\sqrt{x}$. [5 marks]

9. Solve $7^{2x} = 343$. [1 mark]

10. Solve $2^{x-1} \times 3^{2x-3} = 108$. [2 marks]

11. If $3^x = 405$, find 3^{x-4}. [1 mark]

12. Solve $9^{2x} = \frac{1}{27}$. [2 marks]

13. Solve $9^{x-1} = 3^{2x-1} - 54$. [5 marks]

14. If $9^{x+1} = 27^{x-1}$, find 2^x. [3 marks]

15. If $2^x = 3$, $3^y = 5$ and $5^z = 2$, find xyz. [2 marks]

16. If $2^x = 5^y = 20^z$, show that $z = \frac{xy}{x+2y}$. [5 marks]

17. Find the value of $(0.008)^{\frac{1}{3}}$. [1 mark]

18. If $x = 3^{\frac{1}{3}} + 3^{\frac{2}{3}}$, show that $x^3 - 9x = 12$. [3 marks]

19. Find the value of $(243)^{0.13} \times (243)^{0.07}$. [2 marks]

20. Find the value of $\left\{\left(3^{-2}\right)^{3}\right\}^{-\frac{1}{2}}$. [1 mark]

Key sheet

Q	Answer
1	$7^{\frac{4}{5}}$
2	6
3	3
4	96
5	$\frac{89}{18}$
6	4
7	Shown: LHS $= 1 =$ RHS
8	Shown: LHS $= x^{\frac{3}{2}} = x\sqrt{x} =$ RHS
9	$x = \frac{3}{2}$
10	$x = 3$
11	5
12	$x = -\frac{3}{4}$
13	$x = \frac{5}{2}$
14	32
15	$xyz = 1$
16	Shown: $z = \frac{xy}{x+2y}$
17	0.2
18	Shown: $x^3 - 9x = 12$
19	3
20	27

Solutions

1. Write $\sqrt[5]{7^4}$ in exponent form. [1]

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$\sqrt[5]{7^4} = 7^{\frac{4}{5}}$

Ans: $7^{\frac{4}{5}}$

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2. Simplify $\left(\frac{2}{3}\right)^{-3} \times \left(\frac{3}{4}\right)^{-2}$. [2]

$= \left(\frac{3}{2}\right)^{3} \times \left(\frac{4}{3}\right)^{2}$

$= \frac{27}{8} \times \frac{16}{9}$

$= 6$

Ans: 6

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3. Simplify $27^2 \times 3^{-5}$. [1]

$= (3^3)^2 \times 3^{-5}$

$= 3^6 \times 3^{-5}$

$= 3^{6-5} = 3^1$

$= 3$

Ans: 3

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4. Find the value of $\frac{(36)^{\frac{3}{2}} \times (32)^{\frac{2}{5}}}{(27)^{\frac{2}{3}}}$. [2]

$= \frac{(6^2)^{\frac{3}{2}} \times (2^5)^{\frac{2}{5}}}{(3^3)^{\frac{2}{3}}}$

$= \frac{6^3 \times 2^2}{3^2}$

$= \frac{216 \times 4}{9}$

$= 96$

Ans: 96

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5. Find the value of $\left(\frac{27}{64}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{16}{81}\right)^{\frac{1}{4}}} + \frac{\sqrt{25}}{\sqrt[3]{27}}$. [3]

$= \left[\left(\frac{3}{4}\right)^3\right]^{-\frac{2}{3}} + \frac{1}{\left[\left(\frac{2}{3}\right)^4\right]^{\frac{1}{4}}} + \frac{\sqrt{5^2}}{\sqrt[3]{3^3}}$

$= \left(\frac{4}{3}\right)^2 + \frac{1}{\frac{2}{3}} + \frac{5}{3}$

$= \frac{16}{9} + \frac{3}{2} + \frac{5}{3}$

$= \frac{32+27+30}{18}$

$= \frac{89}{18}$

Ans: $\frac{89}{18}$

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6. Simplify $\frac{6^{\frac{1}{5}} \times 9^{-\frac{1}{5}} \times 12}{3^{\frac{8}{5}} \times 15^{-\frac{4}{5}}} \div \frac{5^{\frac{6}{5}} \times 2^{\frac{7}{5}}}{8^{\frac{2}{5}} \times 25^{\frac{1}{5}}}$. [5]

$= \frac{6^{\frac{1}{5}} \times 9^{-\frac{1}{5}} \times 12}{3^{\frac{8}{5}} \times 15^{-\frac{4}{5}}} \times \frac{8^{\frac{2}{5}} \times 25^{\frac{1}{5}}}{5^{\frac{6}{5}} \times 2^{\frac{7}{5}}}$

$= \frac{(2\times3)^{\frac{1}{5}} \times (3^2)^{-\frac{1}{5}} \times (2^2\times3)}{3^{\frac{8}{5}} \times (3\times5)^{-\frac{4}{5}}} \times \frac{(2^3)^{\frac{2}{5}} \times (5^2)^{\frac{1}{5}}}{5^{\frac{6}{5}} \times 2^{\frac{7}{5}}}$

$= \frac{2^{\frac{1}{5}} \times 3^{\frac{1}{5}} \times 3^{-\frac{2}{5}} \times 2^2 \times 3}{3^{\frac{8}{5}} \times 3^{-\frac{4}{5}} \times 5^{-\frac{4}{5}}} \times \frac{2^{\frac{6}{5}} \times 5^{\frac{2}{5}}}{5^{\frac{6}{5}} \times 2^{\frac{7}{5}}}$

$= 2^{\frac{1}{5}+2-\frac{7}{5}+\frac{6}{5}} \times 3^{\frac{1}{5}-\frac{2}{5}+1-\frac{8}{5}+\frac{4}{5}} \times 5^{\frac{4}{5}-\frac{6}{5}+\frac{2}{5}}$

$= 2^2 \times 3^0 \times 5^0$

$= 4$

Ans: 4

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7. Show that $\left(\frac{x^a}{x^b}\right)^c \cdot \left(\frac{x^b}{x^c}\right)^a \cdot \left(\frac{x^c}{x^a}\right)^b = 1$. [3]

LHS $= (x^{a-b})^c \cdot (x^{b-c})^a \cdot (x^{c-a})^b$

$= x^{ac-bc} \cdot x^{ab-ac} \cdot x^{bc-ab}$

$= x^{ac-bc+ab-ac+bc-ab}$

$= x^0 = 1 =$ RHS

Ans: Shown: LHS $= 1 =$ RHS

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8. If $a+b+c=0$, show that $x^{\frac{a^2}{2bc}} \cdot x^{\frac{b^2}{2ca}} \cdot x^{\frac{c^2}{2ab}} = x\sqrt{x}$. [5]

Given $a+b+c=0 \Rightarrow a^3+b^3+c^3 = 3abc$

LHS $= x^{\frac{a^2}{2bc}+\frac{b^2}{2ca}+\frac{c^2}{2ab}}$

$= x^{\frac{a^3+b^3+c^3}{2abc}}$

$= x^{\frac{3abc}{2abc}} = x^{\frac{3}{2}}$

$= x^1 \cdot x^{\frac{1}{2}} = x\sqrt{x} =$ RHS

Ans: Shown: LHS $= x^{\frac{3}{2}} = x\sqrt{x} =$ RHS

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9. Solve $7^{2x} = 343$. [1]

$7^{2x} = 7^3$

$\Rightarrow 2x = 3$

$\Rightarrow x = \frac{3}{2}$

Ans: $x = \frac{3}{2}$

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10. Solve $2^{x-1} \times 3^{2x-3} = 108$. [2]

$108 = 2^2 \times 3^3$

$\Rightarrow 2^{x-1} \times 3^{2x-3} = 2^2 \times 3^3$

$\Rightarrow x-1 = 2$ and $2x-3 = 3$

$\Rightarrow x = 3$ and $x = 3$

$\therefore x = 3$

Ans: $x = 3$

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11. If $3^x = 405$, find 3^{x-4}. [1]

$3^{x-4} = \frac{3^x}{3^4}$

$= \frac{405}{81} = 5$

Ans: 5

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12. Solve $9^{2x} = \frac{1}{27}$. [2]

$(3^2)^{2x} = \frac{1}{3^3}$

$\Rightarrow 3^{4x} = 3^{-3}$

$\Rightarrow 4x = -3$

$\Rightarrow x = -\frac{3}{4}$

Ans: $x = -\frac{3}{4}$

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13. Solve $9^{x-1} = 3^{2x-1} - 54$. [5]

$(3^2)^{x-1} = 3^{2x-1} - 54$

$\Rightarrow 3^{2x-2} = 3^{2x-1} - 54$

$\Rightarrow \frac{3^{2x}}{9} = \frac{3^{2x}}{3} - 54$

$\Rightarrow 3^{2x}\left(\frac{1}{3} - \frac{1}{9}\right) = 54$

$\Rightarrow 3^{2x} \times \frac{2}{9} = 54$

$\Rightarrow 3^{2x} = 54 \times \frac{9}{2} = 243 = 3^5$

$\Rightarrow 2x = 5 \Rightarrow x = \frac{5}{2}$

Ans: $x = \frac{5}{2}$

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14. If $9^{x+1} = 27^{x-1}$, find 2^x. [3]

$(3^2)^{x+1} = (3^3)^{x-1}$

$\Rightarrow 3^{2x+2} = 3^{3x-3}$

$\Rightarrow 2x + 2 = 3x - 3 \Rightarrow x = 5$

$2^x = 2^5 = 32$

Ans: 32

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15. If $2^x = 3$, $3^y = 5$ and $5^z = 2$, find xyz. [2]

$5 = 3^y = (2^x)^y = 2^{xy}$

$2 = 5^z = (2^{xy})^z = 2^{xyz}$

$2^{xyz} = 2 = 2^1$

$\Rightarrow xyz = 1$

Ans: $xyz = 1$

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16. If $2^x = 5^y = 20^z$, show that $z = \frac{xy}{x+2y}$. [5]

Let $2^x = 5^y = 20^z = k$ (say)

$\Rightarrow 2 = k^{\frac{1}{x}}, 5 = k^{\frac{1}{y}}, 20 = k^{\frac{1}{z}}$

$20 = 2^2 \times 5$

$\Rightarrow k^{\frac{1}{z}} = \left(k^{\frac{1}{x}}\right)^2 \times k^{\frac{1}{y}}$

$\Rightarrow k^{\frac{1}{z}} = k^{\frac{2}{x}+\frac{1}{y}}$

$\Rightarrow \frac{1}{z} = \frac{2}{x} + \frac{1}{y}$

$\Rightarrow \frac{1}{z} = \frac{2y+x}{xy}$

$\Rightarrow z = \frac{xy}{x+2y}$

Ans: Shown: $z = \frac{xy}{x+2y}$

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17. Find the value of $(0.008)^{\frac{1}{3}}$. [1]

$0.008 = \frac{8}{1000} = \left(\frac{2}{10}\right)^3 = (0.2)^3$

$(0.008)^{\frac{1}{3}} = \left[(0.2)^3\right]^{\frac{1}{3}} = 0.2$

Ans: 0.2

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18. If $x = 3^{\frac{1}{3}} + 3^{\frac{2}{3}}$, show that $x^3 - 9x = 12$. [3]

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$x^3 = \left(3^{\frac{1}{3}}\right)^3 + \left(3^{\frac{2}{3}}\right)^3 + 3 \cdot 3^{\frac{1}{3}} \cdot 3^{\frac{2}{3}} \left(3^{\frac{1}{3}} + 3^{\frac{2}{3}}\right)$

$= 3 + 9 + 3 \times 3 \times x$

$x^3 = 12 + 9x$

$\Rightarrow x^3 - 9x = 12$

Ans: Shown: $x^3 - 9x = 12$

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19. Find the value of $(243)^{0.13} \times (243)^{0.07}$. [2]

$= (243)^{0.13+0.07} = (243)^{0.2}$

$0.2 = \frac{1}{5}$

$= (3^5)^{\frac{1}{5}} = 3$

Ans: 3

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20. Find the value of $\left\{\left(3^{-2}\right)^3\right\}^{-\frac{1}{2}}$. [1]

$= 3^{(-2)\times 3\times(-\frac{1}{2})}$

$= 3^3$

$= 27$

Ans: 27