

Algebraic Identities

Question Paper, Set 08

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Simplify $(3a + 7b) \times 8a$. [1 mark]
2. Expand $(2m - 7n)(4m - 3n)$. [2 marks]
3. Simplify $(7m + 5n)(7m - 5n)$ using an identity. [1 mark]
4. Find the product $(4m + \frac{1}{3})(4m - \frac{1}{3})$. [2 marks]
5. Find $(8a + 2b)^2$. [1 mark]
6. Expand $(\frac{4x}{5} + \frac{7y}{2})^2$. [2 marks]
7. $(4a - b)^2 = \underline{\hspace{2cm}}$ [1 mark]
8. Simplify $(4 + \frac{1}{x})^2 + (4 - \frac{1}{x})^2$. [2 marks]
9. If $4m + 2n = 20$ and $4m - 2n = 4$, find the value of (i) mn (ii) $16m^2 + 4n^2$. [5 marks]
10. Simplify $(7m + 3n)^2 - 84mn$. [3 marks]
11. If $m + n = 11$ and $mn = 28$, find the value of (i) $m^2 + n^2$ (ii) $m^4 + n^4$. [5 marks]
12. Find $(2m + 3m^2 + \frac{4}{m})^2$. [3 marks]
13. Expand: (i) $(6a - 3b - 2c)^2$ (ii) $(3a + 5b - 7c)^2$ [5 marks]
14. Find $(q + 7)^3$. [1 mark]
15. Find $(4p - q)^3$. [2 marks]
16. If $n + \frac{1}{n} = 11$, find the value of (i) $n^2 + \frac{1}{n^2}$ (ii) $n^3 + \frac{1}{n^3}$. [5 marks]
17. Simplify $(z - 2)(z^2 + 2z + 4)$. [1 mark]
18. If $m - \frac{1}{m} = 8$, find the value of $m^3 - \frac{1}{m^3}$. [3 marks]
19. Using an identity, find the value of $1^3 + 9^3 + 7^3 - 3 \times 1 \times 9 \times 7$. [2 marks]
20. Simplify $(x - 2y)^3 + (2y - 6z)^3 + (6z - x)^3$ without expanding the cubes. [3 marks]

Key sheet

Q	Answer
1	$24a^2 + 56ab$
2	$8m^2 - 34mn + 21n^2$
3	$49m^2 - 25n^2$
4	$16m^2 - \frac{1}{9}$
5	$64a^2 + 4b^2 + 32ab$
6	$\frac{16x^2}{25} + \frac{49y^2}{4} + \frac{28xy}{5}$
7	$16a^2 + b^2 - 8ab$
8	$2(16 + \frac{1}{x^2}) = 32 + \frac{2}{x^2}$
9	(i) $mn = 12$ (ii) $16m^2 + 4n^2 = 208$
10	$(7m - 3n)^2$
11	(i) 65 (ii) 2657
12	$4m^2 + 9m^4 + \frac{16}{m^2} + 12m^3 + 24m + 16$
13	(i) $36a^2 + 9b^2 + 4c^2 - 36ab + 12bc - 24ca$ (ii) $9a^2 + 25b^2 + 49c^2 + 30ab - 70bc - 42ca$
14	$q^3 + 343 + 21q^2 + 147q$
15	$64p^3 - q^3 - 48p^2q + 12pq^2$
16	(i) 119 (ii) 1298
17	$z^3 - 8$
18	536
19	884
20	$3(x - 2y)(2y - 6z)(6z - x)$

Solutions

1. Simplify $(3a + 7b) \times 8a$. [1]

$$(a + b)c = ac + bc$$

$$(3a + 7b) \times 8a = (3a)(8a) + (7b)(8a)$$

$$= 24a^2 + 56ab$$

Ans: $24a^2 + 56ab$

2. Expand $(2m - 7n)(4m - 3n)$. [2]

$$(a - b)(c - d) = ac - ad - bc + bd$$

$$(2m - 7n)(4m - 3n) = (2m)(4m) - (2m)(3n) - (7n)(4m) + (7n)(3n)$$

$$= 8m^2 - 6mn - 28mn + 21n^2$$

$$= 8m^2 - 34mn + 21n^2$$

Ans: $8m^2 - 34mn + 21n^2$

3. Simplify $(7m + 5n)(7m - 5n)$ using an identity. [1]

$$(a + b)(a - b) = a^2 - b^2$$

$$(7m + 5n)(7m - 5n) = (7m)^2 - (5n)^2$$

$$= 49m^2 - 25n^2$$

Ans: $49m^2 - 25n^2$

4. Find the product $(4m + \frac{1}{3})(4m - \frac{1}{3})$. [2]

$$(a + b)(a - b) = a^2 - b^2$$

$$(4m + \frac{1}{3})(4m - \frac{1}{3}) = (4m)^2 - (\frac{1}{3})^2$$

$$= 16m^2 - \frac{1}{9}$$

Ans: $16m^2 - \frac{1}{9}$

5. Find $(8a + 2b)^2$. [1]

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(8a + 2b)^2 = (8a)^2 + (2b)^2 + 2(8a)(2b)$$

$$= 64a^2 + 4b^2 + 32ab$$

Ans: $64a^2 + 4b^2 + 32ab$

6. Expand $(\frac{4x}{5} + \frac{7y}{2})^2$. [2]

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(\frac{4x}{5} + \frac{7y}{2})^2 = (\frac{4x}{5})^2 + (\frac{7y}{2})^2 + 2(\frac{4x}{5})(\frac{7y}{2})$$

$$= \frac{16x^2}{25} + \frac{49y^2}{4} + \frac{28xy}{5}$$

Ans: $\frac{16x^2}{25} + \frac{49y^2}{4} + \frac{28xy}{5}$

7. $(4a - b)^2 = \underline{\hspace{2cm}}$ [1]

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$(4a - b)^2 = (4a)^2 + (b)^2 - 2(4a)(b)$$

$$= 16a^2 + b^2 - 8ab$$

Ans: $16a^2 + b^2 - 8ab$

8. Simplify $(4 + \frac{1}{x})^2 + (4 - \frac{1}{x})^2$. [2]

$$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

$$(4 + \frac{1}{x})^2 + (4 - \frac{1}{x})^2 = 2[(4)^2 + (\frac{1}{x})^2]$$

$$= 2(16 + \frac{1}{x^2})$$

$$= 32 + \frac{2}{x^2}$$

Ans: $2(16 + \frac{1}{x^2}) = 32 + \frac{2}{x^2}$

9. If $4m + 2n = 20$ and $4m - 2n = 4$, find the value of (i) mn (ii) $16m^2 + 4n^2$. [5]

(i) $(a + b)^2 - (a - b)^2 = 4ab$

$$(4m + 2n)^2 - (4m - 2n)^2 = 4(4m)(2n)$$

$$(20)^2 - (4)^2 = 32mn$$

$$400 - 16 = 32mn$$

$$mn = \frac{384}{32} = 12$$

(ii) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

$$(4m + 2n)^2 + (4m - 2n)^2 = 2[(4m)^2 + (2n)^2]$$

$$400 + 16 = 2(16m^2 + 4n^2)$$

$$16m^2 + 4n^2 = \frac{416}{2} = 208$$

Ans: (i) $mn = 12$ (ii) $16m^2 + 4n^2 = 208$

10. Simplify $(7m + 3n)^2 - 84mn$. [3]

$$(a - b)^2 = (a + b)^2 - 4ab$$

Consider $(7m + 3n)^2 - 84mn$

$$= (7m + 3n)^2 - 4(7m)(3n)$$

$$= (7m - 3n)^2$$

Ans: $(7m - 3n)^2$

11. If $m + n = 11$ and $mn = 28$, find the value of (i) $m^2 + n^2$ (ii) $m^4 + n^4$. [5]

$$a^2 + b^2 = (a + b)^2 - 2ab$$

(i) $m^2 + n^2 = (11)^2 - 2(28) = 121 - 56 = 65$

(ii) $m^4 + n^4 = (m^2)^2 + (n^2)^2$

$$= (m^2 + n^2)^2 - 2m^2n^2 = (m^2 + n^2)^2 - 2(mn)^2$$

$$= (65)^2 - 2(28)^2 = 4225 - 1568 = 2657$$

Ans: (i) 65 (ii) 2657

12. Find $(2m + 3m^2 + \frac{4}{m})^2$. [3]

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(2m + 3m^2 + \frac{4}{m})^2 = (2m)^2 + (3m^2)^2 + (\frac{4}{m})^2 + 2(2m)(3m^2) + 2(3m^2)(\frac{4}{m}) + 2(\frac{4}{m})(2m)$$
$$= 4m^2 + 9m^4 + \frac{16}{m^2} + 12m^3 + 24m + 16$$

Ans: $4m^2 + 9m^4 + \frac{16}{m^2} + 12m^3 + 24m + 16$

13. Expand: (i) $(6a - 3b - 2c)^2$ (ii) $(3a + 5b - 7c)^2$ [5]

$$(i) (a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$$

$$(6a - 3b - 2c)^2 = (6a)^2 + (3b)^2 + (2c)^2 - 2(6a)(3b) + 2(3b)(2c) - 2(2c)(6a)$$
$$= 36a^2 + 9b^2 + 4c^2 - 36ab + 12bc - 24ca$$

$$(ii) (a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$$

$$(3a + 5b - 7c)^2 = (3a)^2 + (5b)^2 + (7c)^2 + 2(3a)(5b) - 2(5b)(7c) - 2(7c)(3a)$$
$$= 9a^2 + 25b^2 + 49c^2 + 30ab - 70bc - 42ca$$

Ans: (i) $36a^2 + 9b^2 + 4c^2 - 36ab + 12bc - 24ca$ **(ii)** $9a^2 + 25b^2 + 49c^2 + 30ab - 70bc - 42ca$

14. Find $(q + 7)^3$. [1]

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$(q + 7)^3 = (q)^3 + (7)^3 + 3(q)(7)(q + 7)$$

$$= q^3 + 343 + 21q(q + 7)$$

$$= q^3 + 343 + 21q^2 + 147q$$

Ans: $q^3 + 343 + 21q^2 + 147q$

15. Find $(4p - q)^3$. [2]

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$(4p - q)^3 = (4p)^3 - (q)^3 - 3(4p)(q)(4p - q)$$

$$= 64p^3 - q^3 - 12pq(4p - q)$$

$$= 64p^3 - q^3 - 48p^2q + 12pq^2$$

Ans: $64p^3 - q^3 - 48p^2q + 12pq^2$

16. If $n + \frac{1}{n} = 11$, find the value of (i) $n^2 + \frac{1}{n^2}$ (ii) $n^3 + \frac{1}{n^3}$. [5]

$$(i) a^2 + b^2 = (a + b)^2 - 2ab$$

$$n^2 + \frac{1}{n^2} = (n + \frac{1}{n})^2 - 2 \cdot n \cdot \frac{1}{n}$$

$$= (n + \frac{1}{n})^2 - 2 \cdot n \cdot \frac{1}{n}$$

$$= (11)^2 - 2 = 121 - 2 = 119$$

$$(ii) a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

$$n^3 + \frac{1}{n^3} = (n + \frac{1}{n})^3 - 3 \cdot n \cdot \frac{1}{n} (n + \frac{1}{n})$$

$$= (11)^3 - 3(11) = 1331 - 33 = 1298$$

Ans: (i) 119 **(ii)** 1298

17. Simplify $(z - 2)(z^2 + 2z + 4)$. [1]

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Here $a = z$, $b = 2$

$$(z - 2)(z^2 + 2z + 4) = (z)^3 - (2)^3 = z^3 - 8$$

Ans: $z^3 - 8$

18. If $m - \frac{1}{m} = 8$, find the value of $m^3 - \frac{1}{m^3}$. [3]

$$a^3 - b^3 = (a - b)^3 + 3ab(a - b)$$

Here $a = m$, $b = \frac{1}{m}$, so $ab = m \cdot \frac{1}{m} = 1$

$$\begin{aligned} m^3 - \frac{1}{m^3} &= \left(m - \frac{1}{m}\right)^3 + 3(1)\left(m - \frac{1}{m}\right) \\ &= (8)^3 + 3(8) = 512 + 24 = 536 \end{aligned}$$

Ans: 536

19. Using an identity, find the value of $1^3 + 9^3 + 7^3 - 3 \times 1 \times 9 \times 7$. [2]

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

Here $a = 1$, $b = 9$, $c = 7$

$$a + b + c = 1 + 9 + 7 = 17$$

$$a^2 + b^2 + c^2 - ab - bc - ca = 1 + 81 + 49 - 9 - 63 - 7 = 52$$

$$\text{Value} = 17 \times 52 = 884$$

Ans: 884

20. Simplify $(x - 2y)^3 + (2y - 6z)^3 + (6z - x)^3$ without expanding the cubes. [3]

Take $a = x - 2y$, $b = 2y - 6z$, $c = 6z - x$

$$a + b + c = x - 2y + 2y - 6z + 6z - x = 0$$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

$$\therefore (x - 2y)^3 + (2y - 6z)^3 + (6z - x)^3 = 3(x - 2y)(2y - 6z)(6z - x)$$

Ans: $3(x - 2y)(2y - 6z)(6z - x)$
