

Algebraic Identities

Question Paper, Set 06

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Multiply: $(7m - 8n) \times 6m$. [1 mark]
2. Multiply $(m - 5n)(m + 6n)$. [2 marks]
3. $(4x - 3y)(4x + 3y) = \underline{\hspace{2cm}}$ [1 mark]
4. Multiply $(3r + \frac{1}{9})(3r - \frac{1}{9})$. [2 marks]
5. $(9p + 8q)^2 = \underline{\hspace{2cm}}$ [1 mark]
6. Expand $(\frac{a}{2} + \frac{4b}{5})^2$. [2 marks]
7. Find $(7m - 6n)^2$. [1 mark]
8. Use an identity to simplify $(5 + \frac{6}{x})^2 + (5 - \frac{6}{x})^2$. [2 marks]
9. If $4a + 6b = 26$ and $4a - 6b = 14$, find the value of (i) ab (ii) $16a^2 + 36b^2$. [5 marks]
10. Show that $(4x + 9y)^2 - 144xy = (4x - 9y)^2$. [3 marks]
11. If $p + \frac{1}{p} = 6$, find the value of (i) $p^2 + \frac{1}{p^2}$ (ii) $p^4 + \frac{1}{p^4}$. [5 marks]
12. $(4q + 2q^2 + \frac{3}{q})^2 = \underline{\hspace{2cm}}$ [3 marks]
13. Expand: (i) $(6x - 7y - 2z)^2$ (ii) $(3x - y + 2z)^2$ [5 marks]
14. Find $(r + 6)^3$. [1 mark]
15. Find the value of $(2m - 3)^3$. [2 marks]
16. If $p + \frac{1}{p} = 8$, find the value of (i) $p^2 + \frac{1}{p^2}$ (ii) $p^3 + \frac{1}{p^3}$. [5 marks]
17. Simplify $(q - 5)(q^2 + 5q + 25)$. [1 mark]
18. If $r - \frac{1}{r} = 6$, find the value of $r^3 - \frac{1}{r^3}$. [3 marks]
19. Using an identity, find the value of $5^3 + 1^3 + 7^3 - 3 \times 5 \times 1 \times 7$. [2 marks]

20. Simplify $(p - 5q)^3 + (5q - 3r)^3 + (3r - p)^3$ without expanding the cubes.

[3 marks]

Key sheet

Q	Answer
1	$42m^2 - 48mn$
2	$m^2 + mn - 30n^2$
3	$16x^2 - 9y^2$
4	$9r^2 - \frac{1}{81}$
5	$81p^2 + 64q^2 + 144pq$
6	$\frac{a^2}{4} + \frac{16b^2}{25} + \frac{4ab}{5}$
7	$49m^2 + 36n^2 - 84mn$
8	$2(25 + \frac{36}{x^2}) = 50 + \frac{72}{x^2}$
9	(i) $ab = 5$ (ii) $16a^2 + 36b^2 = 436$
10	$\text{LHS} = (4x - 9y)^2 = \text{RHS}$
11	(i) 34 (ii) 1154
12	$16q^2 + 4q^4 + \frac{9}{q^2} + 16q^3 + 12q + 24$
13	(i) $36x^2 + 49y^2 + 4z^2 - 84xy + 28yz - 24zx$ (ii) $9x^2 + y^2 + 4z^2 - 6xy - 4yz + 12zx$
14	$r^3 + 216 + 18r^2 + 108r$
15	$8m^3 - 27 - 36m^2 + 54m$
16	(i) 62 (ii) 488
17	$q^3 - 125$
18	234
19	364
20	$3(p - 5q)(5q - 3r)(3r - p)$

Solutions

1. Multiply: $(7m - 8n) \times 6m$. [1]

$$(a - b)c = ac - bc$$

$$(7m - 8n) \times 6m = (7m)(6m) - (8n)(6m)$$

$$= 42m^2 - 48mn$$

$$\text{Ans: } 42m^2 - 48mn$$

2. Multiply $(m - 5n)(m + 6n)$. [2]

$$(a - b)(c + d) = ac + ad - bc - bd$$

$$(m - 5n)(m + 6n) = (m)(m) + (m)(6n) - (5n)(m) - (5n)(6n)$$

$$= m^2 + 6mn - 5mn - 30n^2$$

$$= m^2 + mn - 30n^2$$

$$\text{Ans: } m^2 + mn - 30n^2$$

3. $(4x - 3y)(4x + 3y) = \underline{\hspace{2cm}}$ [1]

$$(a + b)(a - b) = a^2 - b^2$$

$$(4x - 3y)(4x + 3y) = (4x)^2 - (3y)^2$$

$$= 16x^2 - 9y^2$$

$$\text{Ans: } 16x^2 - 9y^2$$

4. Multiply $(3r + \frac{1}{9})(3r - \frac{1}{9})$. [2]

$$(a + b)(a - b) = a^2 - b^2$$

$$(3r + \frac{1}{9})(3r - \frac{1}{9}) = (3r)^2 - (\frac{1}{9})^2$$

$$= 9r^2 - \frac{1}{81}$$

$$\text{Ans: } 9r^2 - \frac{1}{81}$$

5. $(9p + 8q)^2 = \underline{\hspace{2cm}}$ [1]

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(9p + 8q)^2 = (9p)^2 + (8q)^2 + 2(9p)(8q)$$

$$= 81p^2 + 64q^2 + 144pq$$

$$\text{Ans: } 81p^2 + 64q^2 + 144pq$$

6. Expand $(\frac{a}{2} + \frac{4b}{5})^2$. [2]

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(\frac{a}{2} + \frac{4b}{5})^2 = (\frac{a}{2})^2 + (\frac{4b}{5})^2 + 2(\frac{a}{2})(\frac{4b}{5})$$

$$= \frac{a^2}{4} + \frac{16b^2}{25} + \frac{4ab}{5}$$

$$\text{Ans: } \frac{a^2}{4} + \frac{16b^2}{25} + \frac{4ab}{5}$$

7. Find $(7m - 6n)^2$. [1]

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$(7m - 6n)^2 = (7m)^2 + (6n)^2 - 2(7m)(6n)$$

$$= 49m^2 + 36n^2 - 84mn$$

$$\text{Ans: } 49m^2 + 36n^2 - 84mn$$

8. Use an identity to simplify $(5 + \frac{6}{x})^2 + (5 - \frac{6}{x})^2$. [2]

$$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

$$(5 + \frac{6}{x})^2 + (5 - \frac{6}{x})^2 = 2[(5)^2 + (\frac{6}{x})^2]$$

$$= 2(25 + \frac{36}{x^2})$$

$$= 50 + \frac{72}{x^2}$$

$$\text{Ans: } 2(25 + \frac{36}{x^2}) = 50 + \frac{72}{x^2}$$

9. If $4a + 6b = 26$ and $4a - 6b = 14$, find the value of (i) ab (ii) $16a^2 + 36b^2$. [5]

$$(i) (a + b)^2 - (a - b)^2 = 4ab$$

$$(4a + 6b)^2 - (4a - 6b)^2 = 4(4a)(6b)$$

$$(26)^2 - (14)^2 = 96ab$$

$$676 - 196 = 96ab$$

$$ab = \frac{480}{96} = 5$$

$$(ii) (a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

$$(4a + 6b)^2 + (4a - 6b)^2 = 2[(4a)^2 + (6b)^2]$$

$$676 + 196 = 2(16a^2 + 36b^2)$$

$$16a^2 + 36b^2 = \frac{872}{2} = 436$$

$$\text{Ans: (i) } ab = 5 \text{ (ii) } 16a^2 + 36b^2 = 436$$

10. Show that $(4x + 9y)^2 - 144xy = (4x - 9y)^2$. [3]

$$(a - b)^2 = (a + b)^2 - 4ab$$

$$\text{LHS} = (4x + 9y)^2 - 144xy$$

$$= (4x + 9y)^2 - 4(4x)(9y)$$

$$= (4x - 9y)^2 = \text{RHS}$$

$$\text{Ans: LHS} = (4x - 9y)^2 = \text{RHS}$$

11. If $p + \frac{1}{p} = 6$, find the value of (i) $p^2 + \frac{1}{p^2}$ (ii) $p^4 + \frac{1}{p^4}$. [5]

$$a^2 + b^2 = (a + b)^2 - 2ab$$

$$(i) p^2 + \frac{1}{p^2} = (p)^2 + \left(\frac{1}{p}\right)^2$$

$$= \left(p + \frac{1}{p}\right)^2 - 2 \cdot p \cdot \frac{1}{p}$$

$$= (6)^2 - 2 = 36 - 2 = 34$$

$$(ii) p^4 + \frac{1}{p^4} = (p^2)^2 + \left(\frac{1}{p^2}\right)^2$$

$$= \left(p^2 + \frac{1}{p^2}\right)^2 - 2 \cdot p^2 \cdot \frac{1}{p^2}$$

$$= (34)^2 - 2 = 1156 - 2 = 1154$$

$$\text{Ans: (i) } 34 \text{ (ii) } 1154$$

12. $\left(4q + 2q^2 + \frac{3}{q}\right)^2 = \underline{\hspace{2cm}}$ [3]

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$\begin{aligned}\left(4q + 2q^2 + \frac{3}{q}\right)^2 &= (4q)^2 + (2q^2)^2 + \left(\frac{3}{q}\right)^2 + 2(4q)(2q^2) + 2(2q^2)\left(\frac{3}{q}\right) + 2\left(\frac{3}{q}\right)(4q) \\ &= 16q^2 + 4q^4 + \frac{9}{q^2} + 16q^3 + 12q + 24\end{aligned}$$

Ans: $16q^2 + 4q^4 + \frac{9}{q^2} + 16q^3 + 12q + 24$

13. Expand: (i) $(6x - 7y - 2z)^2$ (ii) $(3x - y + 2z)^2$ [5]

$$(i) (a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$$

$$\begin{aligned}(6x - 7y - 2z)^2 &= (6x)^2 + (7y)^2 + (2z)^2 - 2(6x)(7y) + 2(7y)(2z) - 2(2z)(6x) \\ &= 36x^2 + 49y^2 + 4z^2 - 84xy + 28yz - 24zx\end{aligned}$$

$$(ii) (a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$$

$$\begin{aligned}(3x - y + 2z)^2 &= (3x)^2 + (y)^2 + (2z)^2 - 2(3x)(y) - 2(y)(2z) + 2(2z)(3x) \\ &= 9x^2 + y^2 + 4z^2 - 6xy - 4yz + 12zx\end{aligned}$$

Ans: (i) $36x^2 + 49y^2 + 4z^2 - 84xy + 28yz - 24zx$ (ii) $9x^2 + y^2 + 4z^2 - 6xy - 4yz + 12zx$

14. Find $(r + 6)^3$. [1]

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$(r + 6)^3 = (r)^3 + (6)^3 + 3(r)(6)(r + 6)$$

$$= r^3 + 216 + 18r(r + 6)$$

$$= r^3 + 216 + 18r^2 + 108r$$

Ans: $r^3 + 216 + 18r^2 + 108r$

15. Find the value of $(2m - 3)^3$. [2]

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$(2m - 3)^3 = (2m)^3 - (3)^3 - 3(2m)(3)(2m - 3)$$

$$= 8m^3 - 27 - 18m(2m - 3)$$

$$= 8m^3 - 27 - 36m^2 + 54m$$

Ans: $8m^3 - 27 - 36m^2 + 54m$

16. If $p + \frac{1}{p} = 8$, find the value of (i) $p^2 + \frac{1}{p^2}$ (ii) $p^3 + \frac{1}{p^3}$. [5]

$$(i) a^2 + b^2 = (a + b)^2 - 2ab$$

$$p^2 + \frac{1}{p^2} = (p)^2 + \left(\frac{1}{p}\right)^2$$

$$= \left(p + \frac{1}{p}\right)^2 - 2 \cdot p \cdot \frac{1}{p}$$

$$= (8)^2 - 2 = 64 - 2 = 62$$

$$(ii) a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

$$p^3 + \frac{1}{p^3} = \left(p + \frac{1}{p}\right)^3 - 3 \cdot p \cdot \frac{1}{p} \left(p + \frac{1}{p}\right)$$

$$= (8)^3 - 3(8) = 512 - 24 = 488$$

Ans: (i) 62 (ii) 488

17. Simplify $(q - 5)(q^2 + 5q + 25)$. [1]

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Here $a = q$, $b = 5$

$$(q - 5)(q^2 + 5q + 25) = (q)^3 - (5)^3 = q^3 - 125$$

Ans: $q^3 - 125$

18. If $r - \frac{1}{r} = 6$, find the value of $r^3 - \frac{1}{r^3}$. [3]

$$a^3 - b^3 = (a - b)^3 + 3ab(a - b)$$

Here $a = r$, $b = \frac{1}{r}$, so $ab = r \cdot \frac{1}{r} = 1$

$$\begin{aligned} r^3 - \frac{1}{r^3} &= \left(r - \frac{1}{r}\right)^3 + 3(1)\left(r - \frac{1}{r}\right) \\ &= (6)^3 + 3(6) = 216 + 18 = 234 \end{aligned}$$

Ans: 234

19. Using an identity, find the value of $5^3 + 1^3 + 7^3 - 3 \times 5 \times 1 \times 7$. [2]

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

Here $a = 5$, $b = 1$, $c = 7$

$$a + b + c = 5 + 1 + 7 = 13$$

$$a^2 + b^2 + c^2 - ab - bc - ca = 25 + 1 + 49 - 5 - 7 - 35 = 28$$

$$\text{Value} = 13 \times 28 = 364$$

Ans: 364

20. Simplify $(p - 5q)^3 + (5q - 3r)^3 + (3r - p)^3$ without expanding the cubes. [3]

Take $a = p - 5q$, $b = 5q - 3r$, $c = 3r - p$

$$a + b + c = p - 5q + 5q - 3r + 3r - p = 0$$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

$$\therefore (p - 5q)^3 + (5q - 3r)^3 + (3r - p)^3 = 3(p - 5q)(5q - 3r)(3r - p)$$

Ans: $3(p - 5q)(5q - 3r)(3r - p)$
