

Algebraic Identities

Question Paper, Set 05

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Simplify $(6p - 3q) \times 9p$. [1 mark]
2. Expand $(4p - 2q)(2p + 7q)$. [2 marks]
3. Simplify $(7x - y)(7x + y)$ using an identity. [1 mark]
4. Find the product $(5a^4 + 5b^2)(5a^4 - 5b^2)$. [2 marks]
5. Find $(3x + y)^2$. [1 mark]
6. Evaluate $(5y + \frac{1}{9})^2$. [2 marks]
7. $(6p - 9q)^2 = \underline{\hspace{2cm}}$ [1 mark]
8. Simplify $(2p + 4q)^2 + (2p - 4q)^2$. [2 marks]
9. If $3x + 4y = 28$ and $3x - 4y = 20$, find the value of (i) xy (ii) $9x^2 + 16y^2$. [5 marks]
10. Simplify $(8p + 6q)^2 - 192pq$. [3 marks]
11. If $r + \frac{1}{r} = 7$, find the value of (i) $r^2 + \frac{1}{r^2}$ (ii) $(r - \frac{1}{r})^2$. [5 marks]
12. Find $(7a + 2b + 6c)^2$. [3 marks]
13. Expand: (i) $(3p - q + 6r)^2$ (ii) $(4p + 7q - 2r)^2$ [5 marks]
14. Expand $(4y + 1)^3$. [1 mark]
15. Find $(4x - 1)^3$. [2 marks]
16. If $m + n = 15$ and $mn = 56$, find the value of (i) $m^2 + n^2$ (ii) $m^3 + n^3$. [5 marks]
17. Find the product $(2x - 3)(4x^2 + 6x + 9)$. [1 mark]
18. If $p - q = 4$ and $pq = 60$, find the value of $p^3 - q^3$. [3 marks]
19. Using an identity, find the product $(2m + 4n + 5)(4m^2 + 16n^2 + 25 - 8mn - 20n - 10m)$. [2 marks]
20. Without finding the cubes, find the value of $12^3 - 7^3 - 5^3$. [3 marks]

Key sheet

Q	Answer
1	$54p^2 - 27pq$
2	$8p^2 + 24pq - 14q^2$
3	$49x^2 - y^2$
4	$25a^8 - 25b^4$
5	$9x^2 + y^2 + 6xy$
6	$25y^2 + \frac{1}{81} + \frac{10y}{9}$
7	$36p^2 + 81q^2 - 108pq$
8	$2(4p^2 + 16q^2) = 8p^2 + 32q^2$
9	(i) $xy = 8$ (ii) $9x^2 + 16y^2 = 592$
10	$(8p - 6q)^2$
11	(i) 47 (ii) 45
12	$49a^2 + 4b^2 + 36c^2 + 28ab + 24bc + 84ca$
13	(i) $9p^2 + q^2 + 36r^2 - 6pq - 12qr + 36rp$ (ii) $16p^2 + 49q^2 + 4r^2 + 56pq - 28qr - 16rp$
14	$64y^3 + 1 + 48y^2 + 12y$
15	$64x^3 - 1 - 48x^2 + 12x$
16	(i) 113 (ii) 855
17	$8x^3 - 27$
18	784
19	$8m^3 + 64n^3 + 125 - 120mn$
20	1260

Solutions

1. Simplify $(6p - 3q) \times 9p$. [1]

$$(a - b)c = ac - bc$$

$$(6p - 3q) \times 9p = (6p)(9p) - (3q)(9p)$$

$$= 54p^2 - 27pq$$

$$\text{Ans: } 54p^2 - 27pq$$

2. Expand $(4p - 2q)(2p + 7q)$. [2]

$$(a - b)(c + d) = ac + ad - bc - bd$$

$$(4p - 2q)(2p + 7q) = (4p)(2p) + (4p)(7q) - (2q)(2p) - (2q)(7q)$$

$$= 8p^2 + 28pq - 4pq - 14q^2$$

$$= 8p^2 + 24pq - 14q^2$$

$$\text{Ans: } 8p^2 + 24pq - 14q^2$$

3. Simplify $(7x - y)(7x + y)$ using an identity. [1]

$$(a + b)(a - b) = a^2 - b^2$$

$$(7x - y)(7x + y) = (7x)^2 - (y)^2$$

$$= 49x^2 - y^2$$

$$\text{Ans: } 49x^2 - y^2$$

4. Find the product $(5a^4 + 5b^2)(5a^4 - 5b^2)$. [2]

$$(a + b)(a - b) = a^2 - b^2$$

$$(5a^4 + 5b^2)(5a^4 - 5b^2) = (5a^4)^2 - (5b^2)^2$$

$$\text{Using } (ab)^n = a^n \times b^n: (5a^4)^2 = 25a^8 \text{ and } (5b^2)^2 = 25b^4$$

$$= 25a^8 - 25b^4$$

$$\text{Ans: } 25a^8 - 25b^4$$

5. Find $(3x + y)^2$. [1]

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(3x + y)^2 = (3x)^2 + (y)^2 + 2(3x)(y)$$

$$= 9x^2 + y^2 + 6xy$$

$$\text{Ans: } 9x^2 + y^2 + 6xy$$

6. Evaluate $(5y + \frac{1}{9})^2$. [2]

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(5y + \frac{1}{9})^2 = (5y)^2 + (\frac{1}{9})^2 + 2(5y)(\frac{1}{9})$$

$$= 25y^2 + \frac{1}{81} + \frac{10y}{9}$$

$$\text{Ans: } 25y^2 + \frac{1}{81} + \frac{10y}{9}$$

7. $(6p - 9q)^2 = \underline{\hspace{2cm}}$ [1]

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$\begin{aligned}(6p - 9q)^2 &= (6p)^2 + (9q)^2 - 2(6p)(9q) \\ &= 36p^2 + 81q^2 - 108pq\end{aligned}$$

Ans: $36p^2 + 81q^2 - 108pq$

8. Simplify $(2p + 4q)^2 + (2p - 4q)^2$. [2]

$$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

$$\begin{aligned}(2p + 4q)^2 + (2p - 4q)^2 &= 2[(2p)^2 + (4q)^2] \\ &= 2(4p^2 + 16q^2) \\ &= 8p^2 + 32q^2\end{aligned}$$

Ans: $2(4p^2 + 16q^2) = 8p^2 + 32q^2$

9. If $3x + 4y = 28$ and $3x - 4y = 20$, find the value of (i) xy (ii) $9x^2 + 16y^2$. [5]

(i) $(a + b)^2 - (a - b)^2 = 4ab$

$$(3x + 4y)^2 - (3x - 4y)^2 = 4(3x)(4y)$$

$$(28)^2 - (20)^2 = 48xy$$

$$784 - 400 = 48xy$$

$$xy = \frac{384}{48} = 8$$

(ii) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

$$(3x + 4y)^2 + (3x - 4y)^2 = 2[(3x)^2 + (4y)^2]$$

$$784 + 400 = 2(9x^2 + 16y^2)$$

$$9x^2 + 16y^2 = \frac{1184}{2} = 592$$

Ans: (i) $xy = 8$ (ii) $9x^2 + 16y^2 = 592$

10. Simplify $(8p + 6q)^2 - 192pq$. [3]

$$(a - b)^2 = (a + b)^2 - 4ab$$

Consider $(8p + 6q)^2 - 192pq$

$$= (8p + 6q)^2 - 4(8p)(6q)$$

$$= (8p - 6q)^2$$

Ans: $(8p - 6q)^2$

11. If $r + \frac{1}{r} = 7$, find the value of (i) $r^2 + \frac{1}{r^2}$ (ii) $\left(r - \frac{1}{r}\right)^2$. [5]

$$a^2 + b^2 = (a + b)^2 - 2ab$$

(i) $r^2 + \frac{1}{r^2} = (r)^2 + \left(\frac{1}{r}\right)^2$

$$= \left(r + \frac{1}{r}\right)^2 - 2 \cdot r \cdot \frac{1}{r}$$

$$= (7)^2 - 2 = 49 - 2 = 47$$

(ii) $(a - b)^2 = a^2 + b^2 - 2ab$

$$\left(r - \frac{1}{r}\right)^2 = r^2 + \frac{1}{r^2} - 2 \cdot r \cdot \frac{1}{r}$$

$$= 47 - 2 = 45$$

Ans: (i) 47 (ii) 45

12. Find $(7a + 2b + 6c)^2$. [3]

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(7a + 2b + 6c)^2 = (7a)^2 + (2b)^2 + (6c)^2 + 2(7a)(2b) + 2(2b)(6c) + 2(6c)(7a) \\ = 49a^2 + 4b^2 + 36c^2 + 28ab + 24bc + 84ca$$

Ans: $49a^2 + 4b^2 + 36c^2 + 28ab + 24bc + 84ca$

13. Expand: (i) $(3p - q + 6r)^2$ (ii) $(4p + 7q - 2r)^2$ [5]

$$(i) (a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$$

$$(3p - q + 6r)^2 = (3p)^2 + (q)^2 + (6r)^2 - 2(3p)(q) - 2(q)(6r) + 2(6r)(3p) \\ = 9p^2 + q^2 + 36r^2 - 6pq - 12qr + 36rp$$

$$(ii) (a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$$

$$(4p + 7q - 2r)^2 = (4p)^2 + (7q)^2 + (2r)^2 + 2(4p)(7q) - 2(7q)(2r) - 2(2r)(4p) \\ = 16p^2 + 49q^2 + 4r^2 + 56pq - 28qr - 16rp$$

Ans: (i) $9p^2 + q^2 + 36r^2 - 6pq - 12qr + 36rp$ **(ii)** $16p^2 + 49q^2 + 4r^2 + 56pq - 28qr - 16rp$

14. Expand $(4y + 1)^3$. [1]

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$(4y + 1)^3 = (4y)^3 + (1)^3 + 3(4y)(1)(4y + 1) \\ = 64y^3 + 1 + 12y(4y + 1) \\ = 64y^3 + 1 + 48y^2 + 12y$$

Ans: $64y^3 + 1 + 48y^2 + 12y$

15. Find $(4x - 1)^3$. [2]

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$(4x - 1)^3 = (4x)^3 - (1)^3 - 3(4x)(1)(4x - 1) \\ = 64x^3 - 1 - 12x(4x - 1) \\ = 64x^3 - 1 - 48x^2 + 12x$$

Ans: $64x^3 - 1 - 48x^2 + 12x$

16. If $m + n = 15$ and $mn = 56$, find the value of (i) $m^2 + n^2$ (ii) $m^3 + n^3$. [5]

$$(i) a^2 + b^2 = (a + b)^2 - 2ab$$

$$m^2 + n^2 = (15)^2 - 2(56) = 225 - 112 = 113$$

$$(ii) a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

$$m^3 + n^3 = (15)^3 - 3(56)(15) \\ = 3375 - 2520 = 855$$

$$\text{Check with } a^3 + b^3 = (a + b)(a^2 - ab + b^2): (15)(113 - 56) = 15 \times 57 = 855$$

Ans: (i) 113 **(ii)** 855

17. Find the product $(2x - 3)(4x^2 + 6x + 9)$. [1]

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\text{Here } a = 2x, b = 3$$

$$(2x - 3)(4x^2 + 6x + 9) = (2x)^3 - (3)^3 = 8x^3 - 27$$

Ans: $8x^3 - 27$

18. If $p - q = 4$ and $pq = 60$, find the value of $p^3 - q^3$. [3]

$$a^3 - b^3 = (a - b)^3 + 3ab(a - b)$$

$$p^3 - q^3 = (4)^3 + 3(60)(4)$$

$$= 64 + 720 = 784$$

Ans: 784

19. Using an identity, find the product $(2m + 4n + 5)(4m^2 + 16n^2 + 25 - 8mn - 20n - 10m)$. [2]

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\text{Here } a = 2m, b = 4n, c = 5$$

$$= (2m)^3 + (4n)^3 + (5)^3 - 3(2m)(4n)(5)$$

$$= 8m^3 + 64n^3 + 125 - 120mn$$

$$\text{Ans: } 8m^3 + 64n^3 + 125 - 120mn$$

20. Without finding the cubes, find the value of $12^3 - 7^3 - 5^3$. [3]

$$\text{Take } a = 12, b = -7, c = -5$$

$$a + b + c = 12 - 7 - 5 = 0$$

$$\text{If } a + b + c = 0, \text{ then } a^3 + b^3 + c^3 = 3abc.$$

$$\therefore 12^3 - 7^3 - 5^3 = 3(12)(-7)(-5) = 1260$$

Ans: 1260
