

Algebraic Identities

Question Paper, Set 10

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Expand $5p(7p - 8q)$. [1 mark]
2. Find the product $(3m - n)(m - 6n)$. [2 marks]
3. Find the product $(5x + 6y)(5x - 6y)$. [1 mark]
4. Find the product $(5m + \frac{1}{8})(5m - \frac{1}{8})$. [2 marks]
5. Expand $(6p + q)^2$. [1 mark]
6. Expand $(\frac{6m}{5} + \frac{3n}{5})^2$. [2 marks]
7. Expand $(7a - 5b)^2$. [1 mark]
8. Simplify $(4 + \frac{5}{p})^2 + (4 - \frac{5}{p})^2$. [2 marks]
9. If $6x + 2y = 10$ and $6x - 2y = 2$, find the value of (i) xy (ii) $36x^2 + 4y^2$. [5 marks]
10. Show that $(4a - 8b)^2 + 128ab = (4a + 8b)^2$. [3 marks]
11. If $p + \frac{1}{p} = 10$, find the value of (i) $p^2 + \frac{1}{p^2}$ (ii) $(p - \frac{1}{p})^2$. [5 marks]
12. Expand $(n^2 + \frac{3}{n} + 4n)^2$. [3 marks]
13. Expand: (i) $(4a + 5b - 3c)^2$ (ii) $(2p - 3q - 5r)^2$ [5 marks]
14. Find $(3t + 1)^3$. [1 mark]
15. Expand $(x - 4y)^3$. [2 marks]
16. If $q + \frac{1}{q} = 12$, find the value of (i) $q^2 + \frac{1}{q^2}$ (ii) $q^3 + \frac{1}{q^3}$. [5 marks]
17. Simplify $(p - 4)(p^2 + 4p + 16)$. [1 mark]
18. If $x - \frac{1}{x} = 11$, find the value of $x^3 - \frac{1}{x^3}$. [3 marks]
19. Using an identity, find the value of $7^3 + 8^3 + 5^3 - 3 \times 7 \times 8 \times 5$. [2 marks]

20. Simplify $(6x - 2y)^3 + (2y - 5z)^3 + (5z - 6x)^3$ without expanding the cubes.

[3 marks]

Key sheet

Q	Answer
1	$35p^2 - 40pq$
2	$3m^2 - 19mn + 6n^2$
3	$25x^2 - 36y^2$
4	$25m^2 - \frac{1}{64}$
5	$36p^2 + q^2 + 12pq$
6	$\frac{36m^2}{25} + \frac{9n^2}{25} + \frac{36mn}{25}$
7	$49a^2 + 25b^2 - 70ab$
8	$2(16 + \frac{25}{p^2}) = 32 + \frac{50}{p^2}$
9	(i) $xy = 2$ (ii) $36x^2 + 4y^2 = 52$
10	$\text{LHS} = (4a + 8b)^2 = \text{RHS}$
11	(i) 98 (ii) 96
12	$n^4 + \frac{9}{n^2} + 16n^2 + 6n + 24 + 8n^3$
13	(i) $16a^2 + 25b^2 + 9c^2 + 40ab - 30bc - 24ca$ (ii) $4p^2 + 9q^2 + 25r^2 - 12pq + 30qr - 20rp$
14	$27t^3 + 1 + 27t^2 + 9t$
15	$x^3 - 64y^3 - 12x^2y + 48xy^2$
16	(i) 142 (ii) 1692
17	$p^3 - 64$
18	1364
19	140
20	$3(6x - 2y)(2y - 5z)(5z - 6x)$

Solutions

1. Expand $5p(7p - 8q)$. [1]

$$a(b - c) = ab - ac$$

$$5p(7p - 8q) = (5p)(7p) - (5p)(8q)$$

$$= 35p^2 - 40pq$$

$$\text{Ans: } 35p^2 - 40pq$$

2. Find the product $(3m - n)(m - 6n)$. [2]

$$(a - b)(c - d) = ac - ad - bc + bd$$

$$(3m - n)(m - 6n) = (3m)(m) - (3m)(6n) - (n)(m) + (n)(6n)$$

$$= 3m^2 - 18mn - mn + 6n^2$$

$$= 3m^2 - 19mn + 6n^2$$

$$\text{Ans: } 3m^2 - 19mn + 6n^2$$

3. Find the product $(5x + 6y)(5x - 6y)$. [1]

$$(a + b)(a - b) = a^2 - b^2$$

$$(5x + 6y)(5x - 6y) = (5x)^2 - (6y)^2$$

$$= 25x^2 - 36y^2$$

$$\text{Ans: } 25x^2 - 36y^2$$

4. Find the product $(5m + \frac{1}{8})(5m - \frac{1}{8})$. [2]

$$(a + b)(a - b) = a^2 - b^2$$

$$(5m + \frac{1}{8})(5m - \frac{1}{8}) = (5m)^2 - (\frac{1}{8})^2$$

$$= 25m^2 - \frac{1}{64}$$

$$\text{Ans: } 25m^2 - \frac{1}{64}$$

5. Expand $(6p + q)^2$. [1]

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(6p + q)^2 = (6p)^2 + (q)^2 + 2(6p)(q)$$

$$= 36p^2 + q^2 + 12pq$$

$$\text{Ans: } 36p^2 + q^2 + 12pq$$

6. Expand $(\frac{6m}{5} + \frac{3n}{5})^2$. [2]

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(\frac{6m}{5} + \frac{3n}{5})^2 = (\frac{6m}{5})^2 + (\frac{3n}{5})^2 + 2(\frac{6m}{5})(\frac{3n}{5})$$

$$= \frac{36m^2}{25} + \frac{9n^2}{25} + \frac{36mn}{25}$$

$$\text{Ans: } \frac{36m^2}{25} + \frac{9n^2}{25} + \frac{36mn}{25}$$

7. Expand $(7a - 5b)^2$. [1]

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$(7a - 5b)^2 = (7a)^2 + (5b)^2 - 2(7a)(5b) \\ = 49a^2 + 25b^2 - 70ab$$

$$\text{Ans: } 49a^2 + 25b^2 - 70ab$$

8. Simplify $\left(4 + \frac{5}{p}\right)^2 + \left(4 - \frac{5}{p}\right)^2$. [2]

$$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

$$\left(4 + \frac{5}{p}\right)^2 + \left(4 - \frac{5}{p}\right)^2 = 2\left[(4)^2 + \left(\frac{5}{p}\right)^2\right]$$

$$= 2\left(16 + \frac{25}{p^2}\right)$$

$$= 32 + \frac{50}{p^2}$$

$$\text{Ans: } 2\left(16 + \frac{25}{p^2}\right) = 32 + \frac{50}{p^2}$$

9. If $6x + 2y = 10$ and $6x - 2y = 2$, find the value of (i) xy (ii) $36x^2 + 4y^2$. [5]

$$(i) (a + b)^2 - (a - b)^2 = 4ab$$

$$(6x + 2y)^2 - (6x - 2y)^2 = 4(6x)(2y)$$

$$(10)^2 - (2)^2 = 48xy$$

$$100 - 4 = 48xy$$

$$xy = \frac{96}{48} = 2$$

$$(ii) (a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

$$(6x + 2y)^2 + (6x - 2y)^2 = 2[(6x)^2 + (2y)^2]$$

$$100 + 4 = 2(36x^2 + 4y^2)$$

$$36x^2 + 4y^2 = \frac{104}{2} = 52$$

$$\text{Ans: (i) } xy = 2 \text{ (ii) } 36x^2 + 4y^2 = 52$$

10. Show that $(4a - 8b)^2 + 128ab = (4a + 8b)^2$. [3]

$$(a + b)^2 = (a - b)^2 + 4ab$$

$$\text{LHS} = (4a - 8b)^2 + 128ab$$

$$= (4a - 8b)^2 + 4(4a)(8b)$$

$$= (4a + 8b)^2 = \text{RHS}$$

$$\text{Ans: LHS} = (4a + 8b)^2 = \text{RHS}$$

11. If $p + \frac{1}{p} = 10$, find the value of (i) $p^2 + \frac{1}{p^2}$ (ii) $\left(p - \frac{1}{p}\right)^2$. [5]

$$a^2 + b^2 = (a + b)^2 - 2ab$$

$$(i) p^2 + \frac{1}{p^2} = (p)^2 + \left(\frac{1}{p}\right)^2$$

$$= \left(p + \frac{1}{p}\right)^2 - 2 \cdot p \cdot \frac{1}{p}$$

$$= (10)^2 - 2 = 100 - 2 = 98$$

$$(ii) (a - b)^2 = a^2 + b^2 - 2ab$$

$$\left(p - \frac{1}{p}\right)^2 = p^2 + \frac{1}{p^2} - 2 \cdot p \cdot \frac{1}{p}$$

$$= 98 - 2 = 96$$

Ans: (i) 98 (ii) 96

12. Expand $\left(n^2 + \frac{3}{n} + 4n\right)^2$. [3]

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$\left(n^2 + \frac{3}{n} + 4n\right)^2 = (n^2)^2 + \left(\frac{3}{n}\right)^2 + (4n)^2 + 2(n^2)\left(\frac{3}{n}\right) + 2\left(\frac{3}{n}\right)(4n) + 2(4n)(n^2)$$

$$= n^4 + \frac{9}{n^2} + 16n^2 + 6n + 24 + 8n^3$$

$$\text{Ans: } n^4 + \frac{9}{n^2} + 16n^2 + 6n + 24 + 8n^3$$

13. Expand: (i) $(4a + 5b - 3c)^2$ (ii) $(2p - 3q - 5r)^2$ [5]

$$(i) (a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$$

$$(4a + 5b - 3c)^2 = (4a)^2 + (5b)^2 + (3c)^2 + 2(4a)(5b) - 2(5b)(3c) - 2(3c)(4a)$$

$$= 16a^2 + 25b^2 + 9c^2 + 40ab - 30bc - 24ca$$

$$(ii) (a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$$

$$(2p - 3q - 5r)^2 = (2p)^2 + (3q)^2 + (5r)^2 - 2(2p)(3q) + 2(3q)(5r) - 2(5r)(2p)$$

$$= 4p^2 + 9q^2 + 25r^2 - 12pq + 30qr - 20rp$$

$$\text{Ans: (i) } 16a^2 + 25b^2 + 9c^2 + 40ab - 30bc - 24ca \text{ (ii) } 4p^2 + 9q^2 + 25r^2 - 12pq + 30qr - 20rp$$

14. Find $(3t + 1)^3$. [1]

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$(3t + 1)^3 = (3t)^3 + (1)^3 + 3(3t)(1)(3t + 1)$$

$$= 27t^3 + 1 + 9t(3t + 1)$$

$$= 27t^3 + 1 + 27t^2 + 9t$$

$$\text{Ans: } 27t^3 + 1 + 27t^2 + 9t$$

15. Expand $(x - 4y)^3$. [2]

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$(x - 4y)^3 = (x)^3 - (4y)^3 - 3(x)(4y)(x - 4y)$$

$$= x^3 - 64y^3 - 12xy(x - 4y)$$

$$= x^3 - 64y^3 - 12x^2y + 48xy^2$$

$$\text{Ans: } x^3 - 64y^3 - 12x^2y + 48xy^2$$

16. If $q + \frac{1}{q} = 12$, find the value of (i) $q^2 + \frac{1}{q^2}$ (ii) $q^3 + \frac{1}{q^3}$. [5]

$$(i) a^2 + b^2 = (a + b)^2 - 2ab$$

$$q^2 + \frac{1}{q^2} = (q)^2 + \left(\frac{1}{q}\right)^2$$

$$= \left(q + \frac{1}{q}\right)^2 - 2 \cdot q \cdot \frac{1}{q}$$

$$= (12)^2 - 2 = 144 - 2 = 142$$

$$(ii) a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

$$q^3 + \frac{1}{q^3} = \left(q + \frac{1}{q}\right)^3 - 3 \cdot q \cdot \frac{1}{q} \left(q + \frac{1}{q}\right)$$

$$= (12)^3 - 3(12) = 1728 - 36 = 1692$$

Ans: (i) 142 (ii) 1692

17. Simplify $(p - 4)(p^2 + 4p + 16)$. [1]

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Here $a = p$, $b = 4$

$$(p - 4)(p^2 + 4p + 16) = (p)^3 - (4)^3 = p^3 - 64$$

Ans: $p^3 - 64$

18. If $x - \frac{1}{x} = 11$, find the value of $x^3 - \frac{1}{x^3}$. [3]

$$a^3 - b^3 = (a - b)^3 + 3ab(a - b)$$

Here $a = x$, $b = \frac{1}{x}$, so $ab = x \cdot \frac{1}{x} = 1$

$$x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3(1) \left(x - \frac{1}{x}\right)$$

$$= (11)^3 + 3(11) = 1331 + 33 = 1364$$

Ans: 1364

19. Using an identity, find the value of $7^3 + 8^3 + 5^3 - 3 \times 7 \times 8 \times 5$. [2]

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

Here $a = 7$, $b = 8$, $c = 5$

$$a + b + c = 7 + 8 + 5 = 20$$

$$a^2 + b^2 + c^2 - ab - bc - ca = 49 + 64 + 25 - 56 - 40 - 35 = 7$$

$$\text{Value} = 20 \times 7 = 140$$

Ans: 140

20. Simplify $(6x - 2y)^3 + (2y - 5z)^3 + (5z - 6x)^3$ without expanding the cubes. [3]

Take $a = 6x - 2y$, $b = 2y - 5z$, $c = 5z - 6x$

$$a + b + c = 6x - 2y + 2y - 5z + 5z - 6x = 0$$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

$$\therefore (6x - 2y)^3 + (2y - 5z)^3 + (5z - 6x)^3 = 3(6x - 2y)(2y - 5z)(5z - 6x)$$

Ans: $3(6x - 2y)(2y - 5z)(5z - 6x)$
