

Algebraic Identities

Question Paper, Set 01

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Expand $4a(4a + 5b)$. [1 mark]
2. Find the product $(5m - 3n)(5m - 5n)$. [2 marks]
3. Find the product $(3p - 6q)(3p + 6q)$. [1 mark]
4. Find the product $(x^5 + 6y^5)(x^5 - 6y^5)$. [2 marks]
5. Expand $(x + 3y)^2$. [1 mark]
6. Evaluate $(5n + \frac{1}{5})^2$. [2 marks]
7. Expand $(2a - 4b)^2$. [1 mark]
8. Simplify $(9a + 7b)^2 + (9a - 7b)^2$. [2 marks]
9. If $6x + y = 27$ and $6x - y = 21$, find the value of (i) xy (ii) $36x^2 + y^2$. [5 marks]
10. Show that $(5x - 2y)^2 + 40xy = (5x + 2y)^2$. [3 marks]
11. If $x + \frac{1}{x} = 3$, find the value of (i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$. [5 marks]
12. Expand $(4x + 7y + z)^2$. [3 marks]
13. Expand: (i) $(p + 2q - 6r)^2$ (ii) $(7x - 6y + 5z)^2$ [5 marks]
14. Expand $(y + 2)^3$. [1 mark]
15. Expand $(3x - 4)^3$. [2 marks]
16. If $p + q = 7$ and $pq = 12$, find the value of (i) $p^2 + q^2$ (ii) $p^3 + q^3$. [5 marks]
17. Find the product $(3t - 5)(9t^2 + 15t + 25)$. [1 mark]
18. If $a - b = 4$ and $ab = 45$, find the value of $a^3 - b^3$. [3 marks]
19. Using an identity, find the product $(2p + 4q + 1)(4p^2 + 16q^2 + 1 - 8pq - 4q - 2p)$. [2 marks]
20. Without finding the cubes, find the value of $6^3 + 9^3 + (-15)^3$. [3 marks]

Key sheet

Q	Answer
1	$16a^2 + 20ab$
2	$25m^2 - 40mn + 15n^2$
3	$9p^2 - 36q^2$
4	$x^{10} - 36y^{10}$
5	$x^2 + 9y^2 + 6xy$
6	$25n^2 + \frac{1}{25} + 2n$
7	$4a^2 + 16b^2 - 16ab$
8	$2(81a^2 + 49b^2) = 162a^2 + 98b^2$
9	(i) $xy = 12$ (ii) $36x^2 + y^2 = 585$
10	$\text{LHS} = (5x + 2y)^2 = \text{RHS}$
11	(i) 7 (ii) 47
12	$16x^2 + 49y^2 + z^2 + 56xy + 14yz + 8zx$
13	(i) $p^2 + 4q^2 + 36r^2 + 4pq - 24qr - 12rp$ (ii) $49x^2 + 36y^2 + 25z^2 - 84xy - 60yz + 70zx$
14	$y^3 + 8 + 6y^2 + 12y$
15	$27x^3 - 64 - 108x^2 + 144x$
16	(i) 25 (ii) 91
17	$27t^3 - 125$
18	604
19	$8p^3 + 64q^3 + 1 - 24pq$
20	-2430

Solutions

1. Expand $4a(4a + 5b)$. [1]

$$a(b + c) = ab + ac$$

$$4a(4a + 5b) = (4a)(4a) + (4a)(5b)$$

$$= 16a^2 + 20ab$$

Ans: $16a^2 + 20ab$

2. Find the product $(5m - 3n)(5m - 5n)$. [2]

$$(a - b)(c - d) = ac - ad - bc + bd$$

$$(5m - 3n)(5m - 5n) = (5m)(5m) - (5m)(5n) - (3n)(5m) + (3n)(5n)$$

$$= 25m^2 - 25mn - 15mn + 15n^2$$

$$= 25m^2 - 40mn + 15n^2$$

Ans: $25m^2 - 40mn + 15n^2$

3. Find the product $(3p - 6q)(3p + 6q)$. [1]

$$(a + b)(a - b) = a^2 - b^2$$

$$(3p - 6q)(3p + 6q) = (3p)^2 - (6q)^2$$

$$= 9p^2 - 36q^2$$

Ans: $9p^2 - 36q^2$

4. Find the product $(x^5 + 6y^5)(x^5 - 6y^5)$. [2]

$$(a + b)(a - b) = a^2 - b^2$$

$$(x^5 + 6y^5)(x^5 - 6y^5) = (x^5)^2 - (6y^5)^2$$

Using $(ab)^n = a^n \times b^n$: $(x^5)^2 = x^{10}$ and $(6y^5)^2 = 36y^{10}$

$$= x^{10} - 36y^{10}$$

Ans: $x^{10} - 36y^{10}$

5. Expand $(x + 3y)^2$. [1]

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(x + 3y)^2 = (x)^2 + (3y)^2 + 2(x)(3y)$$

$$= x^2 + 9y^2 + 6xy$$

Ans: $x^2 + 9y^2 + 6xy$

6. Evaluate $(5n + \frac{1}{5})^2$. [2]

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(5n + \frac{1}{5})^2 = (5n)^2 + (\frac{1}{5})^2 + 2(5n)(\frac{1}{5})$$

$$= 25n^2 + \frac{1}{25} + 2n$$

Ans: $25n^2 + \frac{1}{25} + 2n$

7. Expand $(2a - 4b)^2$. [1]

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$\begin{aligned}(2a - 4b)^2 &= (2a)^2 + (4b)^2 - 2(2a)(4b) \\ &= 4a^2 + 16b^2 - 16ab\end{aligned}$$

Ans: $4a^2 + 16b^2 - 16ab$

8. Simplify $(9a + 7b)^2 + (9a - 7b)^2$. [2]

$$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

$$\begin{aligned}(9a + 7b)^2 + (9a - 7b)^2 &= 2[(9a)^2 + (7b)^2] \\ &= 2(81a^2 + 49b^2) \\ &= 162a^2 + 98b^2\end{aligned}$$

Ans: $2(81a^2 + 49b^2) = 162a^2 + 98b^2$

9. If $6x + y = 27$ and $6x - y = 21$, find the value of (i) xy (ii) $36x^2 + y^2$. [5]

$$(i) (a + b)^2 - (a - b)^2 = 4ab$$

$$(6x + y)^2 - (6x - y)^2 = 4(6x)(y)$$

$$(27)^2 - (21)^2 = 24xy$$

$$729 - 441 = 24xy$$

$$xy = \frac{288}{24} = 12$$

$$(ii) (a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

$$(6x + y)^2 + (6x - y)^2 = 2[(6x)^2 + (y)^2]$$

$$729 + 441 = 2(36x^2 + y^2)$$

$$36x^2 + y^2 = \frac{1170}{2} = 585$$

Ans: (i) $xy = 12$ (ii) $36x^2 + y^2 = 585$

10. Show that $(5x - 2y)^2 + 40xy = (5x + 2y)^2$. [3]

$$(a + b)^2 = (a - b)^2 + 4ab$$

$$\text{LHS} = (5x - 2y)^2 + 40xy$$

$$= (5x - 2y)^2 + 4(5x)(2y)$$

$$= (5x + 2y)^2 = \text{RHS}$$

Ans: $\text{LHS} = (5x + 2y)^2 = \text{RHS}$

11. If $x + \frac{1}{x} = 3$, find the value of (i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$. [5]

$$a^2 + b^2 = (a + b)^2 - 2ab$$

$$(i) x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2$$

$$= (3 + \frac{1}{3})^2 - 2 \cdot 3 \cdot \frac{1}{3}$$

$$= (3)^2 - 2 = 9 - 2 = 7$$

$$(ii) x^4 + \frac{1}{x^4} = (x^2 + \frac{1}{x^2})^2 - 2$$

$$= (7 + \frac{1}{7})^2 - 2 \cdot 7 \cdot \frac{1}{7}$$

$$= (7)^2 - 2 = 49 - 2 = 47$$

Ans: (i) 7 (ii) 47

12. Expand $(4x + 7y + z)^2$. [3]

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(4x + 7y + z)^2 = (4x)^2 + (7y)^2 + (z)^2 + 2(4x)(7y) + 2(7y)(z) + 2(z)(4x) \\ = 16x^2 + 49y^2 + z^2 + 56xy + 14yz + 8zx$$

Ans: $16x^2 + 49y^2 + z^2 + 56xy + 14yz + 8zx$

13. Expand: (i) $(p + 2q - 6r)^2$ (ii) $(7x - 6y + 5z)^2$ [5]

$$(i) (a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$$

$$(p + 2q - 6r)^2 = (p)^2 + (2q)^2 + (6r)^2 + 2(p)(2q) - 2(2q)(6r) - 2(6r)(p) \\ = p^2 + 4q^2 + 36r^2 + 4pq - 24qr - 12rp$$

$$(ii) (a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$$

$$(7x - 6y + 5z)^2 = (7x)^2 + (6y)^2 + (5z)^2 - 2(7x)(6y) - 2(6y)(5z) + 2(5z)(7x) \\ = 49x^2 + 36y^2 + 25z^2 - 84xy - 60yz + 70zx$$

Ans: (i) $p^2 + 4q^2 + 36r^2 + 4pq - 24qr - 12rp$ (ii) $49x^2 + 36y^2 + 25z^2 - 84xy - 60yz + 70zx$

14. Expand $(y + 2)^3$. [1]

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$(y + 2)^3 = (y)^3 + (2)^3 + 3(y)(2)(y + 2)$$

$$= y^3 + 8 + 6y(y + 2)$$

$$= y^3 + 8 + 6y^2 + 12y$$

Ans: $y^3 + 8 + 6y^2 + 12y$

15. Expand $(3x - 4)^3$. [2]

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$(3x - 4)^3 = (3x)^3 - (4)^3 - 3(3x)(4)(3x - 4)$$

$$= 27x^3 - 64 - 36x(3x - 4)$$

$$= 27x^3 - 64 - 108x^2 + 144x$$

Ans: $27x^3 - 64 - 108x^2 + 144x$

16. If $p + q = 7$ and $pq = 12$, find the value of (i) $p^2 + q^2$ (ii) $p^3 + q^3$. [5]

$$(i) a^2 + b^2 = (a + b)^2 - 2ab$$

$$p^2 + q^2 = (7)^2 - 2(12) = 49 - 24 = 25$$

$$(ii) a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

$$p^3 + q^3 = (7)^3 - 3(12)(7)$$

$$= 343 - 252 = 91$$

$$\text{Check with } a^3 + b^3 = (a + b)(a^2 - ab + b^2): (7)(25 - 12) = 7 \times 13 = 91$$

Ans: (i) 25 (ii) 91

17. Find the product $(3t - 5)(9t^2 + 15t + 25)$. [1]

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\text{Here } a = 3t, b = 5$$

$$(3t - 5)(9t^2 + 15t + 25) = (3t)^3 - (5)^3 = 27t^3 - 125$$

Ans: $27t^3 - 125$

18. If $a - b = 4$ and $ab = 45$, find the value of $a^3 - b^3$. [3]

$$a^3 - b^3 = (a - b)^3 + 3ab(a - b)$$

$$a^3 - b^3 = (4)^3 + 3(45)(4)$$

$$= 64 + 540 = 604$$

Ans: 604

19. Using an identity, find the product $(2p + 4q + 1)(4p^2 + 16q^2 + 1 - 8pq - 4q - 2p)$. [2]

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\text{Here } a = 2p, b = 4q, c = 1$$

$$= (2p)^3 + (4q)^3 + (1)^3 - 3(2p)(4q)(1)$$

$$= 8p^3 + 64q^3 + 1 - 24pq$$

$$\text{Ans: } 8p^3 + 64q^3 + 1 - 24pq$$

20. Without finding the cubes, find the value of $6^3 + 9^3 + (-15)^3$. [3]

$$\text{Take } a = 6, b = 9, c = -15$$

$$a + b + c = 6 + 9 - 15 = 0$$

$$\text{If } a + b + c = 0, \text{ then } a^3 + b^3 + c^3 = 3abc.$$

$$\therefore 6^3 + 9^3 + (-15)^3 = 3(6)(9)(-15) = -2430$$

Ans: -2430
