EXPRESSWALA ACADEMY

Trigonometry up to Transformations

Question Paper, Set 09

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\cos\theta = \frac{45}{53}$ and θ is acute, find $\sin\theta$ and $\cot\theta$. [2 marks]

2. Express 250^{g} in degrees. [1 mark]

3. State whether $\tan 100^\circ \sin 300^\circ$ is positive or negative. [1 mark]

4. Evaluate $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$. [2 marks]

5. Simplify $\frac{\sec(90^\circ+\theta)\sin(360^\circ-\theta)\cos(180^\circ+\theta)}{\csc(180^\circ-\theta)\cos(270^\circ-\theta)\sin(270^\circ-\theta)}$. [3 marks]

6. Find the value of $\tan\frac{17\pi}{3}$. [1 mark]

7. Find the greatest value of $3 + 8\sin x$. [1 mark]

8. Find the value of $\sin^2 2^\circ + \sin^2 4^\circ + \sin^2 6^\circ + \cdots + \sin^2 88^\circ$. [3 marks]

9. If $\sin\theta\cos\theta = \frac{2}{7}$, find $\sin^4\theta + \cos^4\theta$. [2 marks]

10. If $\sin A = \frac{12}{37}$ and $\sin B = \frac{4}{5}$, where A and B are acute, find $\cos(A+B)$. [3 marks]

11. If $0 < A < B < \frac{\pi}{4}$, $\tan(A+B) = \frac{20}{21}$ and $\cos(A-B) = \frac{40}{41}$, find $\tan 2B$. [5 marks]

12. In $\triangle ABC$, $\cot A = 4$ and $\cot B = 5$. Find $\cot C$. [2 marks]

13. Find the value of $\tan 18^\circ + \tan 27^\circ + \tan 18^\circ \tan 27^\circ$. [1 mark]

14. If $\cos A = \frac{35}{37}$ and A is acute, find $\cos 2A$ and $\sin 2A$. [2 marks]

15. If $\tan\alpha = -\frac{12}{5}$ and $\frac{\pi}{2} < \alpha < \pi$, find $\cos 3\alpha$ and $\sin 2\alpha$. [5 marks]

16. Find the value of $\csc 18^\circ$. [1 mark]

17. Find the value of $\tan 5^\circ(1+\sec 10^\circ)(1+\sec 20^\circ)$. [3 marks]

18. Prove that $\sin^3\theta + \sin^3(120^\circ+\theta) + \sin^3(240^\circ+\theta) = -\frac{3}{4}\sin 3\theta$. Hence find $\sin^3 10^\circ + \sin^3 130^\circ + \sin^3 250^\circ$. [5 marks]

19. If $A + B + C = 270^\circ$, prove that $\cos 2A + \cos 2B + \cos 2C = 1 - 4\sin A \sin B \sin C$. [5 marks]

20. Write the period of the function $2|\sin x| + 3|\cos x|$. [2 marks]

Key sheet

Q	Answer
1	$\sin\theta = \frac{28}{53}$, $\cot\theta = \frac{45}{28}$
2	225°
3	Positive
4	$\sqrt{3}$
5	-1
6	$-\sqrt{3}$
7	11
8	22
9	$\frac{41}{49}$
10	$\frac{57}{185}$
11	$\frac{989}{660}$
12	$-\frac{19}{9}$
13	1
14	$\cos 2A = \frac{1081}{1369}$, $\sin 2A = \frac{840}{1369}$
15	$\cos 3\alpha = \frac{2035}{2197}$, $\sin 2\alpha = -\frac{120}{169}$
16	$\sqrt{5}+1$
17	$\tan 20^\circ$
18	$-\frac{3}{8}$
19	Proved: LHS $= 1 - 4\sin A \sin B \sin C$
20	π

Solutions

1. If $\cos\theta = \frac{45}{53}$ and θ is acute, find $\sin\theta$ and $\cot\theta$. [2]

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{45}{53}$: take adj $= 45$, hyp $= 53$

opp $= \sqrt{53^2 - 45^2} = \sqrt{784} = 28$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{28}{53}$, $\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{45}{28}$

Ans: $\sin\theta = \frac{28}{53}$, $\cot\theta = \frac{45}{28}$

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2. Express 250^g in degrees. [1]

$200^g = 180^\circ$, so $1^g = \frac{9}{10}$ degree

$250^g = 250 \times \frac{9}{10} = 225^\circ$

Ans: 225°

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3. State whether $\tan 100^\circ \sin 300^\circ$ is positive or negative. [1]

$100^\circ \in \text{II}_Q \Rightarrow \tan 100^\circ$ is -ve

$300^\circ \in \text{IV}_Q \Rightarrow \sin 300^\circ$ is -ve

So the product is positive

Ans: Positive

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4. Evaluate $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$. [2]

$= \frac{2/\sqrt{3}}{1-\frac{1}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \frac{3}{\sqrt{3}}$

$= \sqrt{3}$

Ans: $\sqrt{3}$

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5. Simplify $\frac{\sec(90^\circ+\theta)\sin(360^\circ-\theta)\cos(180^\circ+\theta)}{\csc(180^\circ-\theta)\cos(270^\circ-\theta)\sin(270^\circ-\theta)}$. [3]

Numerator: $\sec(90^\circ+\theta) = -\csc\theta$, $\sin(360^\circ-\theta) = -\sin\theta$, $\cos(180^\circ+\theta) = -\cos\theta$

Denominator: $\csc(180^\circ-\theta) = \csc\theta$, $\cos(270^\circ-\theta) = -\sin\theta$, $\sin(270^\circ-\theta) = -\cos\theta$

G.E $= \frac{(-\csc\theta)(-\sin\theta)(-\cos\theta)}{(\csc\theta)(-\sin\theta)(-\cos\theta)} = -1$

Ans: -1

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6. Find the value of $\tan\frac{17\pi}{3}$. [1]

$\frac{17\pi}{3} = 17 \times 60^\circ = 1020^\circ$

$\tan 1020^\circ = \tan(720^\circ + 300^\circ) = \tan 300^\circ$

$300^\circ \in \text{IV}_Q$, where tan is -ve

$\tan 300^\circ = \tan(360^\circ - 60^\circ) = -\tan 60^\circ = -\sqrt{3}$

Ans: $-\sqrt{3}$

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7. Find the greatest value of $3 + 8\sin x$. [1]

$-1 \le \sin x \le 1$

$\Rightarrow -8 \le 8\sin x \le 8$

$\Rightarrow -5 \le 3 + 8\sin x \le 11$

Greatest value $= 11$ (when $\sin x = 1$)

Ans: 11

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8. Find the value of $\sin^2 2^\circ + \sin^2 4^\circ + \sin^2 6^\circ + \cdots + \sin^2 88^\circ$. [3]

$\sin^2 2^\circ + \sin^2 88^\circ = \sin^2 2^\circ + \cos^2 2^\circ = 1$

There are 44 terms: 22 pairs $(2^\circ, 88^\circ), \ldots, (44^\circ, 46^\circ)$

G.E $= 22(1) = 22$

Ans: 22

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9. If $\sin\theta\cos\theta = \frac{2}{7}$, find $\sin^4\theta + \cos^4\theta$. [2]

$\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta = 1 - 2(\sin\theta\cos\theta)^2$

$= 1 - 2\left(\frac{2}{7}\right)^2 = 1 - \frac{8}{49} = \frac{41}{49}$

Ans: $\frac{41}{49}$

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10. If $\sin A = \frac{12}{37}$ and $\sin B = \frac{4}{5}$, where A and B are acute, find $\cos(A+B)$. [3]

$\sin A = \frac{12}{37} \Rightarrow \cos A = \frac{35}{37}$ (A acute)

$\sin B = \frac{4}{5} \Rightarrow \cos B = \frac{3}{5}$ (B acute)

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$= \frac{35}{37}\cdot\frac{3}{5} - \frac{12}{37}\cdot\frac{4}{5} = \frac{21}{37} - \frac{48}{185} = \frac{57}{185}$

Ans: $\frac{57}{185}$

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11. If $0 < A < B < \frac{\pi}{4}$, $\tan(A+B) = \frac{20}{21}$ and $\cos(A-B) = \frac{40}{41}$, find $\tan 2B$. [5]

$0 < A < B < \frac{\pi}{4} \Rightarrow 0 < A+B < \frac{\pi}{2}$ and $A - B < 0$

$\tan(A+B) = \frac{20}{21}$

$\cos(A-B) = \frac{40}{41}$ and $A - B < 0 \Rightarrow \sin(A-B) = -\frac{9}{41}$, $\tan(A-B) = -\frac{9}{40}$

$\tan 2B = \tan[(A+B) - (A-B)] = \frac{\tan(A+B) - \tan(A-B)}{1 + \tan(A+B)\tan(A-B)}$

$= \frac{\frac{20}{21} + \frac{9}{40}}{1 - \frac{20}{21}\cdot\frac{9}{40}} = \frac{\frac{989}{840}}{\frac{11}{14}}$

$= \frac{989}{660}$

Ans: $\frac{989}{660}$

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12. In $\triangle ABC$, $\cot A = 4$ and $\cot B = 5$. Find $\cot C$. [2]

In $\triangle ABC$: $\cot A\cot B + \cot B\cot C + \cot C\cot A = 1$

$20 + (4+5)\cot C = 1$

$9\cot C = 1 - 20 \Rightarrow \cot C = -\frac{19}{9}$

Ans: $-\frac{19}{9}$

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13. Find the value of $\tan 18^\circ + \tan 27^\circ + \tan 18^\circ\tan 27^\circ$. [1]

$18^\circ + 27^\circ = 45^\circ \Rightarrow \frac{\tan 18^\circ + \tan 27^\circ}{1 - \tan 18^\circ\tan 27^\circ} = 1$

$\Rightarrow \tan 18^\circ + \tan 27^\circ + \tan 18^\circ\tan 27^\circ = 1$

Ans: 1

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14. If $\cos A = \frac{35}{37}$ and A is acute, find $\cos 2A$ and $\sin 2A$. [2]

$\cos A = \frac{35}{37} \Rightarrow \sin A = \frac{12}{37}$, $\tan A = \frac{12}{35}$ (A acute)

$\cos 2A = 2\cos^2 A - 1 = 2\cdot\frac{1225}{1369} - 1 = \frac{1081}{1369}$

$\sin 2A = 2\sin A\cos A = 2\cdot\frac{12}{37}\cdot\frac{35}{37} = \frac{840}{1369}$

Ans: $\cos 2A = \frac{1081}{1369}$, $\sin 2A = \frac{840}{1369}$

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15. If $\tan\alpha = -\frac{12}{5}$ and $\frac{\pi}{2} < \alpha < \pi$, find $\cos 3\alpha$ and $\sin 2\alpha$. [5]

Given $\tan\alpha = -\frac{12}{5}$ and $\alpha \in \text{II}_Q$

Right triangle with sides 12, 5, 13; with the signs of II_Q: $\sin\alpha = \frac{12}{13}$, $\cos\alpha = -\frac{5}{13}$, $\tan\alpha = -\frac{12}{5}$

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha = 4\left(-\frac{5}{13}\right)^3 - 3\left(-\frac{5}{13}\right) = -\frac{500}{2197} + \frac{15}{13} = \frac{2035}{2197}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\left(\frac{12}{13}\right)\left(-\frac{5}{13}\right) = -\frac{120}{169}$

Ans: $\cos 3\alpha = \frac{2035}{2197}$, $\sin 2\alpha = -\frac{120}{169}$

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16. Find the value of $\csc 18^\circ$. [1]

$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

$\csc 18^\circ = \frac{4}{\sqrt{5}-1} = \frac{4(\sqrt{5}+1)}{5-1} = \sqrt{5}+1$

Ans: $\sqrt{5}+1$

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17. Find the value of $\tan 5^\circ(1+\sec 10^\circ)(1+\sec 20^\circ)$. [3]

$\tan\frac{\theta}{2}(1+\sec\theta) = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} \cdot \frac{2\cos^2\frac{\theta}{2}}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

$\tan 5^\circ(1+\sec 10^\circ) = \tan 10^\circ$

$\tan 10^\circ(1+\sec 20^\circ) = \tan 20^\circ$

Ans: $\tan 20^\circ$

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18. Prove that $\sin^3\theta + \sin^3(120^\circ+\theta) + \sin^3(240^\circ+\theta) = -\frac{3}{4}\sin 3\theta$. Hence find $\sin^3 10^\circ + \sin^3 130^\circ + \sin^3 250^\circ$. [5]

Let $a = \sin\theta$, $b = \sin(120^\circ+\theta)$, $c = \sin(240^\circ+\theta)$

$b + c = 2\sin(180^\circ+\theta)\cos 60^\circ = -\sin\theta$, so $a+b+c = 0$ and $a^3+b^3+c^3 = 3abc$

$b = \sin(60^\circ-\theta)$, $c = -\sin(60^\circ+\theta)$, so $abc = -\sin\theta\sin(60^\circ-\theta)\sin(60^\circ+\theta) = -\frac{1}{4}\sin 3\theta$

LHS $= 3abc = -\frac{3}{4}\sin 3\theta$ = RHS

Put $\theta = 10^\circ$: value $= -\frac{3}{4}\sin 30^\circ = -\frac{3}{8}$

Ans: $-\frac{3}{8}$

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19. If $A+B+C = 270^\circ$, prove that $\cos 2A + \cos 2B + \cos 2C = 1 - 4\sin A\sin B\sin C$. [5]

$A + B = 270^\circ - C \Rightarrow \cos(A+B) = \cos(270^\circ - C) = -\sin C$

LHS $= 2\cos(A+B)\cos(A-B) + 1 - 2\sin^2 C = 1 - 2\sin C[\cos(A-B) + \sin C]$

$= 1 - 2\sin C[\cos(A-B) - \cos(A+B)] = 1 - 2\sin C \cdot 2\sin A\sin B = 1 - 4\sin A\sin B\sin C$

= RHS

Ans: Proved: LHS $= 1 - 4\sin A\sin B\sin C$

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20. Write the period of the function $2|\sin x| + 3|\cos x|$. [2]

For $a|\sin x| + b|\cos x|$ the period is π when $a \neq b$

Here $a = 2 \neq b = 3$, so period $= \pi$

Ans: π