

# Trigonometry up to Transformations

## Question Paper, Set 07

Time: 1 hour 30 minutes

Maximum marks: 50

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If  $\sin \theta = \frac{12}{37}$  and  $\theta$  is acute, find  $\sec \theta$  and  $\cot \theta$ . [2 marks]
2. Express  $\frac{13\pi}{18}$  radians in degrees. [1 mark]
3. State whether  $\csc 320^\circ \cos 140^\circ$  is positive or negative. [1 mark]
4. Evaluate  $\sec^2 60^\circ - \tan^2 60^\circ + \sin^2 90^\circ$ . [2 marks]
5. Simplify  $\frac{\sec(270^\circ - \theta) \cos(-\theta) \cot(180^\circ + \theta)}{\csc(720^\circ + \theta) \sin(90^\circ - \theta) \tan(90^\circ - \theta)}$ . [3 marks]
6. Find the value of  $\cos \frac{25\pi}{6}$ . [1 mark]
7. Find the least value of  $6 - 2 \cos x$ . [1 mark]
8. Find the value of  $\cos^2 3^\circ + \cos^2 6^\circ + \cos^2 9^\circ + \dots + \cos^2 87^\circ$ . [3 marks]
9. If  $\sin \theta \cos \theta = \frac{3}{8}$ , find  $\sin^6 \theta + \cos^6 \theta$ . [2 marks]
10. If  $\sin A = \frac{9}{41}$  and  $\cos B = \frac{4}{5}$ , where  $A$  and  $B$  are acute, find  $\sin(A + B)$ . [3 marks]
11. If  $0 < A < B < \frac{\pi}{4}$ ,  $\sin(A + B) = \frac{8}{17}$  and  $\cos(A - B) = \frac{12}{13}$ , find  $\tan 2B$ . [5 marks]
12. In  $\triangle ABC$ ,  $\tan A = 2$  and  $\tan B = 5$ . Find  $\tan C$ . [2 marks]
13. Find the value of  $(\cot 40^\circ - 1)(\cot 5^\circ - 1)$ . [1 mark]
14. If  $\tan A = \frac{12}{5}$  and  $A$  is acute, find  $\sin 2A$  and  $\cos 2A$ . [2 marks]
15. If  $\sin \alpha = -\frac{4}{5}$  and  $\pi < \alpha < \frac{3\pi}{2}$ , find  $\cos 3\alpha$  and  $\tan 2\alpha$ . [5 marks]
16. Using  $\sin \frac{A}{2} = \sqrt{\frac{1 - \cos A}{2}}$ , find  $\sin 22\frac{1}{2}^\circ$ . [1 mark]
17. Find the value of  $\cos \frac{\pi}{17} \cos \frac{2\pi}{17} \cos \frac{4\pi}{17} \cos \frac{8\pi}{17}$ . [3 marks]
18. Prove that  $\tan \theta + \tan(60^\circ + \theta) + \tan(120^\circ + \theta) = 3 \tan 3\theta$ . Hence find  $\tan 10^\circ + \tan 70^\circ + \tan 130^\circ$ . [5 marks]
19. In  $\triangle ABC$ , prove that  $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} - \sin^2 \frac{C}{2} = 1 - 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$ . [5 marks]

**20.** Find the period of  $\sin 2x + \cos 3x$ .

[2 marks]

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## Key sheet

| Q  | Answer                                                                          |
|----|---------------------------------------------------------------------------------|
| 1  | $\sec \theta = \frac{37}{35}, \cot \theta = \frac{35}{12}$                      |
| 2  | $130^\circ$                                                                     |
| 3  | Positive                                                                        |
| 4  | 2                                                                               |
| 5  | -1                                                                              |
| 6  | $\frac{\sqrt{3}}{2}$                                                            |
| 7  | 4                                                                               |
| 8  | $\frac{29}{2}$                                                                  |
| 9  | $\frac{37}{64}$                                                                 |
| 10 | $\frac{156}{205}$                                                               |
| 11 | $\frac{171}{140}$                                                               |
| 12 | $\frac{7}{9}$                                                                   |
| 13 | 2                                                                               |
| 14 | $\sin 2A = \frac{120}{169}, \cos 2A = -\frac{119}{169}$                         |
| 15 | $\cos 3\alpha = \frac{117}{125}, \tan 2\alpha = -\frac{24}{7}$                  |
| 16 | $\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$                                           |
| 17 | $\frac{1}{16}$                                                                  |
| 18 | $\sqrt{3}$                                                                      |
| 19 | Proved: $\text{LHS} = 1 - 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$ |
| 20 | $2\pi$                                                                          |

## Solutions

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1. If  $\sin \theta = \frac{12}{37}$  and  $\theta$  is acute, find  $\sec \theta$  and  $\cot \theta$ . [2]

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{12}{37}: \text{ take opp} = 12, \text{ hyp} = 37$$

$$\text{adj} = \sqrt{37^2 - 12^2} = \sqrt{1225} = 35$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{37}{35}, \cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{35}{12}$$

$$\text{Ans: } \sec \theta = \frac{37}{35}, \cot \theta = \frac{35}{12}$$

2. Express  $\frac{13\pi}{18}$  radians in degrees. [1]

$$\pi = 180^\circ$$

$$\frac{13\pi}{18} = \frac{13}{18} \times 180^\circ = 130^\circ$$

$$\text{Ans: } 130^\circ$$

3. State whether  $\csc 320^\circ \cos 140^\circ$  is positive or negative. [1]

$$320^\circ \in \text{IV}_Q \Rightarrow \csc 320^\circ \text{ is -ve}$$

$$140^\circ \in \text{II}_Q \Rightarrow \cos 140^\circ \text{ is -ve}$$

So the product is positive

$$\text{Ans: Positive}$$

4. Evaluate  $\sec^2 60^\circ - \tan^2 60^\circ + \sin^2 90^\circ$ . [2]

$$= (2)^2 - (\sqrt{3})^2 + (1)^2$$

$$= 4 - 3 + 1$$

$$= 2$$

$$\text{Ans: } 2$$

5. Simplify  $\frac{\sec(270^\circ - \theta) \cos(-\theta) \cot(180^\circ + \theta)}{\csc(720^\circ + \theta) \sin(90^\circ - \theta) \tan(90^\circ - \theta)}$ . [3]

$$\text{Numerator: } \sec(270^\circ - \theta) = -\csc \theta, \cos(-\theta) = \cos \theta, \cot(180^\circ + \theta) = \cot \theta$$

$$\text{Denominator: } \csc(720^\circ + \theta) = \csc \theta, \sin(90^\circ - \theta) = \cos \theta, \tan(90^\circ - \theta) = \cot \theta$$

$$\text{G.E} = \frac{(-\csc \theta)(\cos \theta)(\cot \theta)}{(\csc \theta)(\cos \theta)(\cot \theta)} = -1$$

$$\text{Ans: } -1$$

6. Find the value of  $\cos \frac{25\pi}{6}$ . [1]

$$\frac{25\pi}{6} = 25 \times 30^\circ = 750^\circ$$

$$\cos 750^\circ = \cos(720^\circ + 30^\circ) = \cos 30^\circ$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\text{Ans: } \frac{\sqrt{3}}{2}$$

7. Find the least value of  $6 - 2 \cos x$ . [1]

$$-1 \leq \cos x \leq 1$$

$$\Rightarrow -2 \leq -2 \cos x \leq 2$$

$$\Rightarrow 4 \leq 6 - 2 \cos x \leq 8$$

$$\text{Least value} = 4 \text{ (when } \cos x = 1)$$

$$\text{Ans: } 4$$

8. Find the value of  $\cos^2 3^\circ + \cos^2 6^\circ + \cos^2 9^\circ + \dots + \cos^2 87^\circ$ . [3]

$$\cos^2 3^\circ + \cos^2 87^\circ = \cos^2 3^\circ + \sin^2 3^\circ = 1$$

There are 29 terms: 14 pairs and the middle term  $\cos^2 45^\circ$

$$\text{G.E} = 14(1) + \frac{1}{2} = \frac{29}{2}$$

**Ans:**  $\frac{29}{2}$

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9. If  $\sin \theta \cos \theta = \frac{3}{8}$ , find  $\sin^6 \theta + \cos^6 \theta$ . [2]

$$\begin{aligned}\sin^6 \theta + \cos^6 \theta &= (\sin^2 \theta + \cos^2 \theta)^3 - 3 \sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta) = 1 - 3(\sin \theta \cos \theta)^2 \\ &= 1 - 3 \left(\frac{3}{8}\right)^2 = 1 - \frac{27}{64} = \frac{37}{64}\end{aligned}$$

**Ans:**  $\frac{37}{64}$

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10. If  $\sin A = \frac{9}{41}$  and  $\cos B = \frac{4}{5}$ , where  $A$  and  $B$  are acute, find  $\sin(A + B)$ . [3]

$$\sin A = \frac{9}{41} \Rightarrow \cos A = \frac{40}{41} \text{ (A acute)}$$

$$\cos B = \frac{4}{5} \Rightarrow \sin B = \frac{3}{5} \text{ (B acute)}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{9}{41} \cdot \frac{4}{5} + \frac{40}{41} \cdot \frac{3}{5} = \frac{36}{205} + \frac{24}{41} = \frac{156}{205}$$

**Ans:**  $\frac{156}{205}$

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11. If  $0 < A < B < \frac{\pi}{4}$ ,  $\sin(A + B) = \frac{8}{17}$  and  $\cos(A - B) = \frac{12}{13}$ , find  $\tan 2B$ . [5]

$$0 < A < B < \frac{\pi}{4} \Rightarrow 0 < A + B < \frac{\pi}{2} \text{ and } A - B < 0$$

$$\sin(A + B) = \frac{8}{17} \Rightarrow \cos(A + B) = \frac{15}{17}, \tan(A + B) = \frac{8}{15}$$

$$\cos(A - B) = \frac{12}{13} \text{ and } A - B < 0 \Rightarrow \sin(A - B) = -\frac{5}{13}, \tan(A - B) = -\frac{5}{12}$$

$$\tan 2B = \tan[(A + B) - (A - B)] = \frac{\tan(A+B) - \tan(A-B)}{1 + \tan(A+B)\tan(A-B)}$$

$$= \frac{\frac{8}{15} + \frac{5}{12}}{1 - \frac{8}{15} \cdot \frac{5}{12}} = \frac{\frac{19}{20}}{\frac{7}{9}}$$

$$= \frac{171}{140}$$

**Ans:**  $\frac{171}{140}$

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12. In  $\triangle ABC$ ,  $\tan A = 2$  and  $\tan B = 5$ . Find  $\tan C$ . [2]

$$\text{In } \triangle ABC: \tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$2 + 5 + \tan C = 10 \tan C$$

$$(10 - 1) \tan C = 7 \Rightarrow \tan C = \frac{7}{9}$$

**Ans:**  $\frac{7}{9}$

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13. Find the value of  $(\cot 40^\circ - 1)(\cot 5^\circ - 1)$ . [1]

$$(\cot 40^\circ - 1)(\cot 5^\circ - 1) = (1 - \cot 40^\circ)(1 - \cot 5^\circ)$$

$$40^\circ + 5^\circ = 45^\circ, \text{ so the value is } 2$$

**Ans:** 2

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14. If  $\tan A = \frac{12}{5}$  and  $A$  is acute, find  $\sin 2A$  and  $\cos 2A$ . [2]

$$\tan A = \frac{12}{5}, \tan^2 A = \frac{144}{25}$$

$$\sin 2A = \frac{2 \tan A}{1 + \tan^2 A} = \frac{\frac{24}{5}}{\frac{169}{25}} = \frac{120}{169}$$

$$\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{-\frac{119}{25}}{\frac{169}{25}} = -\frac{119}{169}$$

$$\text{Ans: } \sin 2A = \frac{120}{169}, \cos 2A = -\frac{119}{169}$$

15. If  $\sin \alpha = -\frac{4}{5}$  and  $\pi < \alpha < \frac{3\pi}{2}$ , find  $\cos 3\alpha$  and  $\tan 2\alpha$ . [5]

$$\text{Given } \sin \alpha = -\frac{4}{5} \text{ and } \alpha \in \text{III}_Q$$

$$\text{Right triangle with sides 4, 3, 5; with the signs of III}_Q: \sin \alpha = -\frac{4}{5}, \cos \alpha = -\frac{3}{5}, \tan \alpha = \frac{4}{3}$$

$$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha = 4 \left(-\frac{3}{5}\right)^3 - 3 \left(-\frac{3}{5}\right) = -\frac{108}{125} + \frac{9}{5} = \frac{117}{125}$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{\frac{8}{3}}{1 - \frac{16}{9}} = \frac{\frac{8}{3}}{-\frac{7}{9}} = -\frac{24}{7}$$

$$\text{Ans: } \cos 3\alpha = \frac{117}{125}, \tan 2\alpha = -\frac{24}{7}$$

16. Using  $\sin \frac{A}{2} = \sqrt{\frac{1 - \cos A}{2}}$ , find  $\sin 22\frac{1}{2}^\circ$ . [1]

$$\text{Take } A = 45^\circ: \sin 22\frac{1}{2}^\circ = \sqrt{\frac{1 - \frac{1}{\sqrt{2}}}{2}}$$

$$= \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

$$\text{Ans: } \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

17. Find the value of  $\cos \frac{\pi}{17} \cos \frac{2\pi}{17} \cos \frac{4\pi}{17} \cos \frac{8\pi}{17}$ . [3]

$$\cos \theta \cos 2\theta \cos 4\theta \cos 8\theta = \frac{\sin 16\theta}{16 \sin \theta} \text{ with } \theta = \frac{\pi}{17}$$

$$\text{G.E} = \frac{\sin \frac{16\pi}{17}}{16 \sin \frac{\pi}{17}}$$

$$\sin \frac{16\pi}{17} = \sin \left(\pi - \frac{\pi}{17}\right) = \sin \frac{\pi}{17}$$

$$\text{G.E} = \frac{1}{16}$$

$$\text{Ans: } \frac{1}{16}$$

18. Prove that  $\tan \theta + \tan(60^\circ + \theta) + \tan(120^\circ + \theta) = 3 \tan 3\theta$ . Hence find  $\tan 10^\circ + \tan 70^\circ + \tan 130^\circ$ . [5]

$$\tan(120^\circ + \theta) = \tan(180^\circ - (60^\circ - \theta)) = -\tan(60^\circ - \theta)$$

$$\text{With } t = \tan \theta: \tan(60^\circ + \theta) - \tan(60^\circ - \theta) = \frac{\sqrt{3}+t}{1-\sqrt{3}t} - \frac{\sqrt{3}-t}{1+\sqrt{3}t} = \frac{8t}{1-3t^2}$$

$$\text{LHS} = t + \frac{8t}{1-3t^2} = \frac{9t-3t^3}{1-3t^2} = 3 \cdot \frac{3t-t^3}{1-3t^2} = 3 \tan 3\theta = \text{RHS}$$

$$\text{Put } \theta = 10^\circ: \tan 10^\circ + \tan 70^\circ + \tan 130^\circ = 3 \tan 30^\circ = \sqrt{3}$$

$$\text{Ans: } \sqrt{3}$$

19. In  $\triangle ABC$ , prove that  $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} - \sin^2 \frac{C}{2} = 1 - 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$ . [5]

$$\frac{A+B}{2} = 90^\circ - \frac{C}{2}, \text{ so } \sin \frac{A+B}{2} = \cos \frac{C}{2} \text{ and } \cos \frac{A+B}{2} = \sin \frac{C}{2}$$

$$\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} = 1 - \left(\cos^2 \frac{A}{2} - \sin^2 \frac{B}{2}\right) = 1 - \cos \frac{A+B}{2} \cos \frac{A-B}{2} = 1 - \sin \frac{C}{2} \cos \frac{A-B}{2}$$

$$\text{LHS} = 1 - \sin \frac{C}{2} \left[\cos \frac{A-B}{2} + \sin \frac{C}{2}\right] = 1 - \sin \frac{C}{2} \left[\cos \frac{A-B}{2} + \cos \frac{A+B}{2}\right]$$

$$= 1 - \sin \frac{C}{2} \cdot 2 \cos \frac{A}{2} \cos \frac{B}{2}$$

$$= \text{RHS}$$

$$\text{Ans: Proved: LHS} = 1 - 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$$

**20.** Find the period of  $\sin 2x + \cos 3x$ . [2]

Period of  $\sin 2x = \pi$ , period of  $\cos 3x = \frac{2\pi}{3}$

$$\text{Period} = \text{L.C.M}\left\{\frac{\pi}{1}, \frac{2\pi}{3}\right\} = \frac{\text{L.C.M}\{\pi, 2\pi\}}{\text{G.C.D}\{1, 3\}} = 2\pi$$

**Ans:**  $2\pi$

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