EXPRESSWALA ACADEMY

Trigonometry up to Transformations

Question Paper, Set 10

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\sec\theta = \frac{65}{56}$ and θ is acute, find $\tan\theta$ and $\csc\theta$. [2 marks]

2. Express 165° in radians. [1 mark]

3. Evaluate $\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ$. [2 marks]

4. Simplify $\frac{\csc(180^\circ+\theta)\csc(270^\circ+\theta)\tan(90^\circ+\theta)}{\sec(90^\circ+\theta)\sec(180^\circ+\theta)\cot(360^\circ-\theta)}$. [3 marks]

5. Find the value of $\sec\frac{17\pi}{4}$. [1 mark]

6. Find the range of $1 - 2\sin x$. [1 mark]

7. Which is greater: $\sin 3$ or $\sin 3^\circ$? [1 mark]

8. Find the value of $\cos^2 15^\circ + \cos^2 25^\circ + \cos^2 35^\circ + \cos^2 45^\circ + \cos^2 55^\circ + \cos^2 65^\circ + \cos^2 75^\circ$. [3 marks]

9. Find the least value of $9\cos^2\theta + \sec^2\theta$. [2 marks]

10. If $\tan A = \frac{5}{12}$ and $\cos B = \frac{24}{25}$, where A and B are acute, find $\sin(A+B)$. [3 marks]

11. If $0 < A < B < \frac{\pi}{4}$, $\sin(A+B) = \frac{3}{5}$ and $\sin(A-B) = -\frac{8}{17}$, find $\tan 2A$. [5 marks]

12. Find the value of $\cos^2 48^\circ - \sin^2 12^\circ$. [1 mark]

13. In $\triangle ABC$, $\tan A = 1$ and $\tan B = 4$. Find $\tan C$. [2 marks]

14. If $\sin A = \frac{11}{61}$ and A is acute, find $\sin 2A$ and $\cos 2A$. [2 marks]

15. If $\sin\alpha = \frac{9}{41}$ and $\frac{\pi}{2} < \alpha < \pi$, find $\cos 3\alpha$ and $\tan 2\alpha$. [5 marks]

16. Find the value of $\cos\frac{2\pi}{9} + \cos\frac{4\pi}{9} + \cos\frac{6\pi}{9} + \cos\frac{8\pi}{9}$. [3 marks]

17. Prove that $\tan(60^\circ+\theta)\tan(60^\circ-\theta) + \tan(60^\circ-\theta)\tan\theta - \tan\theta\tan(60^\circ+\theta) = 3$. Hence find $\tan 65^\circ\tan 55^\circ + \tan 55^\circ\tan 5^\circ - \tan 5^\circ\tan 65^\circ$. [5 marks]

18. If $\frac{x}{y} = \frac{\sin 105^\circ}{\sin 45^\circ}$, find $\frac{x+y}{x-y}$. [1 mark]

19. If $A + B + C = 2S$, prove that $\sin(S-A) + \sin(S-B) + \sin(S-C) - \sin S = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$. [5 marks]

20. Find the period of $\sin 5x + \tan 3x$. [2 marks]

Key sheet

Q	Answer
1	$\tan\theta = \frac{33}{56}$, $\csc\theta = \frac{65}{33}$
2	$\frac{11\pi}{12}$
3	1
4	1
5	$\sqrt{2}$
6	$[-1, 3]$
7	$\sin 3$ is greater
8	$\frac{7}{2}$
9	6
10	$\frac{204}{325}$
11	$\frac{13}{84}$
12	$\frac{\sqrt{5}+1}{8}$
13	$\frac{5}{3}$
14	$\sin 2A = \frac{1320}{3721}$, $\cos 2A = \frac{3479}{3721}$
15	$\cos 3\alpha = -\frac{54280}{68921}$, $\tan 2\alpha = -\frac{720}{1519}$
16	$-\frac{1}{2}$
17	3
18	$3 + 2\sqrt{3}$
19	Proved: LHS $= 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
20	2π

Solutions

1. If $\sec\theta = \frac{65}{56}$ and θ is acute, find $\tan\theta$ and $\csc\theta$. [2]

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{65}{56}$: take hyp $= 65$, adj $= 56$

$\text{opp} = \sqrt{65^2 - 56^2} = \sqrt{1089} = 33$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{33}{56}$, $\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{65}{33}$

Ans: $\tan\theta = \frac{33}{56}$, $\csc\theta = \frac{65}{33}$

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2. Express 165° in radians. [1]

$180^\circ = \pi$, so $1^\circ = \frac{\pi}{180}$

$165^\circ = 165 \times \frac{\pi}{180} = \frac{11\pi}{12}$

Ans: $\frac{11\pi}{12}$

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3. Evaluate $\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ$. [2]

$= (\sqrt{3})^2 - 2\left(\frac{1}{2}\right)^2 - \frac{3}{4}(\sqrt{2})^2$

$= 3 - \frac{1}{2} - \frac{3}{2}$

$= 1$

Ans: 1

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4. Simplify $\frac{\csc(180^\circ+\theta)\csc(270^\circ+\theta)\tan(90^\circ+\theta)}{\sec(90^\circ+\theta)\sec(180^\circ+\theta)\cot(360^\circ-\theta)}$. [3]

Numerator: $\csc(180^\circ+\theta) = -\csc\theta$, $\csc(270^\circ+\theta) = -\sec\theta$, $\tan(90^\circ+\theta) = -\cot\theta$

Denominator: $\sec(90^\circ+\theta) = -\csc\theta$, $\sec(180^\circ+\theta) = -\sec\theta$, $\cot(360^\circ-\theta) = -\cot\theta$

G.E $= \frac{(-\csc\theta)(-\sec\theta)(-\cot\theta)}{(-\csc\theta)(-\sec\theta)(-\cot\theta)} = 1$

Ans: 1

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5. Find the value of $\sec\frac{17\pi}{4}$. [1]

$\frac{17\pi}{4} = 17 \times 45^\circ = 765^\circ$

$\sec 765^\circ = \sec(720^\circ + 45^\circ) = \sec 45^\circ$

$\sec 45^\circ = \sqrt{2}$

Ans: $\sqrt{2}$

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6. Find the range of $1 - 2\sin x$. [1]

$-1 \le \sin x \le 1$

$\Rightarrow -2 \le -2\sin x \le 2$

$\Rightarrow -1 \le 1 - 2\sin x \le 3$

Range $= [-1, 3]$

Ans: $[-1, 3]$

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7. Which is greater: $\sin 3$ or $\sin 3^\circ$? [1]

$3 = 3^c \approx 171^\circ$

$\sin 171^\circ = \sin(180^\circ - 9^\circ) = \sin 9^\circ$

$3^\circ < 9^\circ \Rightarrow \sin 3^\circ < \sin 3$

Ans: $\sin 3$ **is greater**

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8. Find the value of $\cos^2 15^\circ + \cos^2 25^\circ + \cos^2 35^\circ + \cos^2 45^\circ + \cos^2 55^\circ + \cos^2 65^\circ + \cos^2 75^\circ$. [3]

$\cos^2 15^\circ + \cos^2 75^\circ = \cos^2 15^\circ + \sin^2 15^\circ = 1$; similarly for $(25^\circ, 65^\circ)$, $(35^\circ, 55^\circ)$

G.E $= 3(1) + \cos^2 45^\circ = 3 + \frac{1}{2} = \frac{7}{2}$

Ans: $\frac{7}{2}$

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9. Find the least value of $9\cos^2\theta + \sec^2\theta$. [2]

Consider $9\cos^2\theta$ and $\sec^2\theta$; both are +ve

A.M $\geq$ G.M: $\frac{9\cos^2\theta + \sec^2\theta}{2} \geq \sqrt{9\cos^2\theta \cdot \sec^2\theta} = \sqrt{9} = 3$

$\Rightarrow 9\cos^2\theta + \sec^2\theta \geq 6$; equality when $\cos^2\theta = \frac{1}{3}$

Least value $= 6$

Ans: 6

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10. If $\tan A = \frac{5}{12}$ and $\cos B = \frac{24}{25}$, where A and B are acute, find $\sin(A+B)$. [3]

$\tan A = \frac{5}{12} \Rightarrow \sin A = \frac{5}{13}, \cos A = \frac{12}{13}$

$\cos B = \frac{24}{25} \Rightarrow \sin B = \frac{7}{25}$ (B acute)

$\sin(A+B) = \sin A\cos B + \cos A \sin B$

$= \frac{5}{13}\cdot\frac{24}{25} + \frac{12}{13}\cdot\frac{7}{25} = \frac{24}{65} + \frac{84}{325} = \frac{204}{325}$

Ans: $\frac{204}{325}$

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11. If $0 < A < B < \frac{\pi}{4}$, $\sin(A+B) = \frac{3}{5}$ and $\sin(A-B) = -\frac{8}{17}$, find $\tan 2A$. [5]

$0 < A < B < \frac{\pi}{4} \Rightarrow 0 < A+B < \frac{\pi}{2}$ and $A - B < 0$

$\sin(A+B) = \frac{3}{5} \Rightarrow \cos(A+B) = \frac{4}{5}, \tan(A+B) = \frac{3}{4}$

$\sin(A-B) = -\frac{8}{17} \Rightarrow \cos(A-B) = \frac{15}{17}, \tan(A-B) = -\frac{8}{15}$

$\tan 2A = \tan[(A+B) + (A-B)] = \frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B)\tan(A-B)}$

$= \frac{\frac{3}{4} - \frac{8}{15}}{1 + \frac{3}{4}\cdot\frac{8}{15}} = \frac{\frac{13}{60}}{\frac{7}{5}}$

$= \frac{13}{84}$

Ans: $\frac{13}{84}$

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12. Find the value of $\cos^2 48^\circ - \sin^2 12^\circ$. [1]

$\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$

$= \cos 60^\circ \cos 36^\circ$

$= \frac{1}{2}\cdot\frac{\sqrt{5}+1}{4}$

$= \frac{\sqrt{5}+1}{8}$

Ans: $\frac{\sqrt{5}+1}{8}$

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13. In $\triangle ABC$, $\tan A = 1$ and $\tan B = 4$. Find $\tan C$. [2]

In $\triangle ABC$: $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$1 + 4 + \tan C = 4\tan C$

$(4-1)\tan C = 5 \Rightarrow \tan C = \frac{5}{3}$

Ans: $\frac{5}{3}$

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14. If $\sin A = \frac{11}{61}$ and A is acute, find $\sin 2A$ and $\cos 2A$. [2]

$\sin A = \frac{11}{61} \Rightarrow \cos A = \frac{60}{61}$, $\tan A = \frac{11}{60}$ (A acute)

$\sin 2A = 2\sin A\cos A = 2 \cdot \frac{11}{61} \cdot \frac{60}{61} = \frac{1320}{3721}$

$\cos 2A = 1 - 2\sin^2 A = 1 - 2 \cdot \frac{121}{3721} = \frac{3479}{3721}$

Ans: $\sin 2A = \frac{1320}{3721}$, $\cos 2A = \frac{3479}{3721}$

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15. If $\sin\alpha = \frac{9}{41}$ and $\frac{\pi}{2} < \alpha < \pi$, find $\cos 3\alpha$ and $\tan 2\alpha$. [5]

Given $\sin\alpha = \frac{9}{41}$ and $\alpha \in \text{II}_Q$

Right triangle with sides 9, 40, 41; with the signs of II_Q: $\sin\alpha = \frac{9}{41}$, $\cos\alpha = -\frac{40}{41}$, $\tan\alpha = -\frac{9}{40}$

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha = 4\left(-\frac{40}{41}\right)^3 - 3\left(-\frac{40}{41}\right) = -\frac{256000}{68921} + \frac{120}{41} = -\frac{54280}{68921}$

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{-\frac{9}{20}}{1-\frac{81}{1600}} = \frac{-\frac{9}{20}}{\frac{1519}{1600}} = -\frac{720}{1519}$

Ans: $\cos 3\alpha = -\frac{54280}{68921}$, $\tan 2\alpha = -\frac{720}{1519}$

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16. Find the value of $\cos\frac{2\pi}{9} + \cos\frac{4\pi}{9} + \cos\frac{6\pi}{9} + \cos\frac{8\pi}{9}$. [3]

Angles in A.P.: $a_1 = \frac{2\pi}{9}$, $a_n = \frac{8\pi}{9}$, $d = \frac{2\pi}{9}$, $n = 4$

Sum $= \frac{\cos\frac{a_1+a_n}{2}\sin\frac{nd}{2}}{\sin\frac{d}{2}}$

$= \frac{\cos\frac{5\pi}{9}\sin\frac{4\pi}{9}}{\sin\frac{\pi}{9}}$, and $\cos\frac{5\pi}{9} = -\cos\frac{4\pi}{9}$

$= -\frac{\sin\frac{8\pi}{9}}{2\sin\frac{\pi}{9}} = -\frac{1}{2}$

Ans: $-\frac{1}{2}$

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17. Prove that $\tan(60^\circ+\theta)\tan(60^\circ-\theta) + \tan(60^\circ-\theta)\tan\theta - \tan\theta\tan(60^\circ+\theta) = 3$. Hence find $\tan 65^\circ\tan 55^\circ + \tan 55^\circ\tan 5^\circ - \tan 5^\circ\tan 65^\circ$. [5]

With $t = \tan\theta$: $\tan(60^\circ+\theta)\tan(60^\circ-\theta) = \frac{3-t^2}{1-3t^2}$

$\tan(60^\circ+\theta) - \tan(60^\circ-\theta) = \frac{8t}{1-3t^2}$, so $t[\tan(60^\circ-\theta) - \tan(60^\circ+\theta)] = -\frac{8t^2}{1-3t^2}$

LHS $= \frac{3-t^2-8t^2}{1-3t^2} = \frac{3(1-3t^2)}{1-3t^2} = 3$ = RHS

Put $\theta = 5^\circ$: the value is 3

Ans: 3

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18. If $\frac{x}{y} = \frac{\sin 105^\circ}{\sin 45^\circ}$, find $\frac{x+y}{x-y}$. [1]

$\frac{x}{y} = \frac{\sin A}{\sin B} \Rightarrow \frac{x+y}{x-y} = \frac{\sin A+\sin B}{\sin A-\sin B} = \frac{\tan\frac{A+B}{2}}{\tan\frac{A-B}{2}}$

$= \frac{\tan 75^\circ}{\tan 30^\circ} = (2+\sqrt{3})\sqrt{3} = 2\sqrt{3}+3$

Ans: $3+2\sqrt{3}$

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19. If $A+B+C = 2S$, prove that $\sin(S-A) + \sin(S-B) + \sin(S-C) - \sin S = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$. [5]

$\sin(S-A) + \sin(S-B) = 2\sin\frac{2S-A-B}{2}\cos\frac{B-A}{2} = 2\sin\frac{C}{2}\cos\frac{A-B}{2}$

$\sin(S-C) - \sin S = 2\cos\frac{2S-C}{2}\sin\left(-\frac{C}{2}\right) = -2\cos\frac{A+B}{2}\sin\frac{C}{2}$

LHS $= 2\sin\frac{C}{2}\left[\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right] = 2\sin\frac{C}{2} \cdot 2\sin\frac{A}{2}\sin\frac{B}{2}$

= RHS

Ans: Proved: LHS $= 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

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20. Find the period of $\sin 5x + \tan 3x$. [2]

Period of $\sin 5x = \frac{2\pi}{5}$, period of $\tan 3x = \frac{\pi}{3}$

Period $=$ L.C.M$\left\{\frac{2\pi}{5}, \frac{\pi}{3}\right\} = \frac{\text{L.C.M}\{2\pi,\pi\}}{\text{G.C.D}\{5,3\}} = 2\pi$

Ans: 2π