

Trigonometry up to Transformations

Question Paper, Set 01

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\sin \theta = \frac{3}{5}$ and θ is acute, find $\cos \theta$ and $\tan \theta$. [2 marks]
2. Express 75° in radians. [1 mark]
3. State whether $\sec 290^\circ$ is positive or negative. [1 mark]
4. Evaluate $4 \sin^2 30^\circ + 3 \tan^2 60^\circ - 2 \sec^2 45^\circ$. [2 marks]
5. Simplify $\frac{\cos(180^\circ + \theta) \tan(180^\circ + \theta) \csc(-\theta)}{\sin(90^\circ + \theta) \cot(270^\circ - \theta) \sec(90^\circ + \theta)}$. [3 marks]
6. Find the value of $\sin 1140^\circ$. [1 mark]
7. Find the greatest value of $7 - 3 \cos x$. [1 mark]
8. Find the value of $\sin^2 1^\circ + \sin^2 2^\circ + \sin^2 3^\circ + \dots + \sin^2 89^\circ$. [3 marks]
9. If $\sin \theta \cos \theta = \frac{1}{3}$, find $\sin^4 \theta + \cos^4 \theta$. [2 marks]
10. If $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$, where A and B are acute, find $\sin(A + B)$. [3 marks]
11. If $0 < A < B < \frac{\pi}{4}$, $\sin(A + B) = \frac{7}{25}$ and $\cos(A - B) = \frac{40}{41}$, find $\tan 2A$. [5 marks]
12. In $\triangle ABC$, $\tan A = 2$ and $\tan B = 3$. Find $\tan C$. [2 marks]
13. Find the value of $(1 + \tan 13^\circ)(1 + \tan 32^\circ)$. [1 mark]
14. If $\tan A = \frac{3}{4}$ and A is acute, find $\sin 2A$ and $\cos 2A$. [2 marks]
15. If $\sin \alpha = \frac{5}{13}$ and $\frac{\pi}{2} < \alpha < \pi$, find $\sin 3\alpha$ and $\tan 2\alpha$. [5 marks]
16. Find the value of $\cos 36^\circ - \sin 18^\circ$. [1 mark]
17. Find the value of $\cos \frac{\pi}{9} \cos \frac{2\pi}{9} \cos \frac{4\pi}{9}$. [3 marks]
18. Prove that $\cos \theta \cos(60^\circ - \theta) \cos(60^\circ + \theta) = \frac{1}{4} \cos 3\theta$. Hence find the value of $\cos 10^\circ \cos 50^\circ \cos 70^\circ$. [5 marks]
19. In $\triangle ABC$, prove that $\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C$. [5 marks]

20. Find the period of $\sin \frac{x}{3} + \cos \frac{x}{4}$.

[2 marks]

Key sheet

Q	Answer
1	$\cos \theta = \frac{4}{5}, \tan \theta = \frac{3}{4}$
2	$\frac{5\pi}{12}$
3	Positive
4	6
5	-1
6	$\frac{\sqrt{3}}{2}$
7	10
8	$\frac{89}{2}$
9	$\frac{7}{9}$
10	$\frac{56}{65}$
11	$\frac{64}{1023}$
12	1
13	2
14	$\sin 2A = \frac{24}{25}, \cos 2A = \frac{7}{25}$
15	$\sin 3\alpha = \frac{2035}{2197}, \tan 2\alpha = -\frac{120}{119}$
16	$\frac{1}{2}$
17	$\frac{1}{8}$
18	$\frac{\sqrt{3}}{8}$
19	Proved: $\text{LHS} = 4 \cos A \cos B \sin C$
20	24π

Solutions

1. If $\sin \theta = \frac{3}{5}$ and θ is acute, find $\cos \theta$ and $\tan \theta$. [2]

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}: \text{take opp} = 3, \text{hyp} = 5$$

$$\text{adj} = \sqrt{5^2 - 3^2} = \sqrt{16} = 4$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}, \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$$

$$\text{Ans: } \cos \theta = \frac{4}{5}, \tan \theta = \frac{3}{4}$$

2. Express 75° in radians. [1]

$$180^\circ = \pi, \text{ so } 1^\circ = \frac{\pi}{180}$$

$$75^\circ = 75 \times \frac{\pi}{180} = \frac{5\pi}{12}$$

$$\text{Ans: } \frac{5\pi}{12}$$

3. State whether $\sec 290^\circ$ is positive or negative. [1]

$$290^\circ \in \text{IV}_Q \Rightarrow \sec 290^\circ \text{ is +ve}$$

$$\text{Ans: Positive}$$

4. Evaluate $4 \sin^2 30^\circ + 3 \tan^2 60^\circ - 2 \sec^2 45^\circ$. [2]

$$= 4 \left(\frac{1}{2}\right)^2 + 3(\sqrt{3})^2 - 2(\sqrt{2})^2$$

$$= 1 + 9 - 4$$

$$= 6$$

$$\text{Ans: } 6$$

5. Simplify $\frac{\cos(180^\circ + \theta) \tan(180^\circ + \theta) \csc(-\theta)}{\sin(90^\circ + \theta) \cot(270^\circ - \theta) \sec(90^\circ + \theta)}$. [3]

$$\text{Numerator: } \cos(180^\circ + \theta) = -\cos \theta, \tan(180^\circ + \theta) = \tan \theta, \csc(-\theta) = -\csc \theta$$

$$\text{Denominator: } \sin(90^\circ + \theta) = \cos \theta, \cot(270^\circ - \theta) = \tan \theta, \sec(90^\circ + \theta) = -\csc \theta$$

$$\text{G.E} = \frac{(-\cos \theta)(\tan \theta)(-\csc \theta)}{(\cos \theta)(\tan \theta)(-\csc \theta)} = -1$$

$$\text{Ans: } -1$$

6. Find the value of $\sin 1140^\circ$. [1]

$$\sin 1140^\circ = \sin(1080^\circ + 60^\circ) = \sin 60^\circ$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\text{Ans: } \frac{\sqrt{3}}{2}$$

7. Find the greatest value of $7 - 3 \cos x$. [1]

$$-1 \leq \cos x \leq 1$$

$$\Rightarrow -3 \leq -3 \cos x \leq 3$$

$$\Rightarrow 4 \leq 7 - 3 \cos x \leq 10$$

$$\text{Greatest value} = 10 \text{ (when } \cos x = -1)$$

$$\text{Ans: } 10$$

8. Find the value of $\sin^2 1^\circ + \sin^2 2^\circ + \sin^2 3^\circ + \dots + \sin^2 89^\circ$. [3]

$$1^\circ + 89^\circ = 90^\circ: \sin^2 1^\circ + \sin^2 89^\circ = \sin^2 1^\circ + \cos^2 1^\circ = 1$$

The 89 terms make 44 such pairs, and $\sin^2 45^\circ$ is left

$$\text{G.E} = 44(1) + \left(\frac{1}{\sqrt{2}}\right)^2 = 44 + \frac{1}{2} = \frac{89}{2}$$

Ans: $\frac{89}{2}$

9. If $\sin \theta \cos \theta = \frac{1}{3}$, find $\sin^4 \theta + \cos^4 \theta$. [2]

$$\sin^4 \theta + \cos^4 \theta = (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta = 1 - 2(\sin \theta \cos \theta)^2$$

$$= 1 - 2\left(\frac{1}{3}\right)^2 = 1 - \frac{2}{9} = \frac{7}{9}$$

Ans: $\frac{7}{9}$

10. If $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$, where A and B are acute, find $\sin(A + B)$. [3]

$$\sin A = \frac{3}{5} \Rightarrow \cos A = \frac{4}{5} \text{ (A acute)}$$

$$\cos B = \frac{12}{13} \Rightarrow \sin B = \frac{5}{13} \text{ (B acute)}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{3}{5} \cdot \frac{12}{13} + \frac{4}{5} \cdot \frac{5}{13} = \frac{36}{65} + \frac{4}{13} = \frac{56}{65}$$

Ans: $\frac{56}{65}$

11. If $0 < A < B < \frac{\pi}{4}$, $\sin(A + B) = \frac{7}{25}$ and $\cos(A - B) = \frac{40}{41}$, find $\tan 2A$. [5]

$$0 < A < B < \frac{\pi}{4} \Rightarrow 0 < A + B < \frac{\pi}{2} \text{ and } A - B < 0$$

$$\sin(A + B) = \frac{7}{25} \Rightarrow \cos(A + B) = \frac{24}{25}, \tan(A + B) = \frac{7}{24}$$

$$\cos(A - B) = \frac{40}{41} \text{ and } A - B < 0 \Rightarrow \sin(A - B) = -\frac{9}{41}, \tan(A - B) = -\frac{9}{40}$$

$$\tan 2A = \tan[(A + B) + (A - B)] = \frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B)\tan(A-B)}$$

$$= \frac{\frac{7}{24} - \frac{9}{40}}{1 + \frac{7}{24} \cdot \frac{9}{40}} = \frac{\frac{15}{320}}{\frac{311}{320}}$$

$$= \frac{64}{1023}$$

Ans: $\frac{64}{1023}$

12. In $\triangle ABC$, $\tan A = 2$ and $\tan B = 3$. Find $\tan C$. [2]

$$\text{In } \triangle ABC: \tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$2 + 3 + \tan C = 6 \tan C$$

$$(6 - 1) \tan C = 5 \Rightarrow \tan C = 1$$

Ans: 1

13. Find the value of $(1 + \tan 13^\circ)(1 + \tan 32^\circ)$. [1]

$$13^\circ + 32^\circ = 45^\circ$$

$$\text{If } A + B = 45^\circ: \tan(A + B) = 1 \Rightarrow \tan A + \tan B + \tan A \tan B = 1$$

$$\Rightarrow 1 + \tan A + \tan B + \tan A \tan B = 2 \Rightarrow (1 + \tan A)(1 + \tan B) = 2$$

Ans: 2

14. If $\tan A = \frac{3}{4}$ and A is acute, find $\sin 2A$ and $\cos 2A$. [2]

$$\tan A = \frac{3}{4}, \tan^2 A = \frac{9}{16}$$

$$\sin 2A = \frac{2 \tan A}{1 + \tan^2 A} = \frac{\frac{3}{2}}{\frac{25}{16}} = \frac{24}{25}$$

$$\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

$$\text{Ans: } \sin 2A = \frac{24}{25}, \cos 2A = \frac{7}{25}$$

15. If $\sin \alpha = \frac{5}{13}$ and $\frac{\pi}{2} < \alpha < \pi$, find $\sin 3\alpha$ and $\tan 2\alpha$. [5]

$$\text{Given } \sin \alpha = \frac{5}{13} \text{ and } \alpha \in \Pi_Q$$

$$\text{Right triangle with sides 5, 12, 13; with the signs of } \Pi_Q: \sin \alpha = \frac{5}{13}, \cos \alpha = -\frac{12}{13}, \tan \alpha = -\frac{5}{12}$$

$$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha = 3 \left(\frac{5}{13} \right) - 4 \left(\frac{5}{13} \right)^3 = \frac{15}{13} - \frac{500}{2197} = \frac{2035}{2197}$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{-\frac{5}{6}}{1 - \frac{25}{144}} = \frac{-\frac{5}{6}}{\frac{119}{144}} = -\frac{120}{119}$$

$$\text{Ans: } \sin 3\alpha = \frac{2035}{2197}, \tan 2\alpha = -\frac{120}{119}$$

16. Find the value of $\cos 36^\circ - \sin 18^\circ$. [1]

$$\cos 36^\circ = \frac{\sqrt{5}+1}{4}, \sin 18^\circ = \frac{\sqrt{5}-1}{4}$$

$$\cos 36^\circ - \sin 18^\circ = \frac{(\sqrt{5}+1) - (\sqrt{5}-1)}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\text{Ans: } \frac{1}{2}$$

17. Find the value of $\cos \frac{\pi}{9} \cos \frac{2\pi}{9} \cos \frac{4\pi}{9}$. [3]

$$\cos \theta \cos 2\theta \cos 4\theta = \frac{\sin 8\theta}{8 \sin \theta} \text{ with } \theta = \frac{\pi}{9}$$

$$\text{G.E} = \frac{\sin \frac{8\pi}{9}}{8 \sin \frac{\pi}{9}}$$

$$\sin \frac{8\pi}{9} = \sin \left(\pi - \frac{\pi}{9} \right) = \sin \frac{\pi}{9}$$

$$\text{G.E} = \frac{1}{8}$$

$$\text{Ans: } \frac{1}{8}$$

18. Prove that $\cos \theta \cos(60^\circ - \theta) \cos(60^\circ + \theta) = \frac{1}{4} \cos 3\theta$. Hence find the value of $\cos 10^\circ \cos 50^\circ \cos 70^\circ$. [5]

$$\cos(60^\circ + \theta) \cos(60^\circ - \theta) = \cos^2 60^\circ - \sin^2 \theta = \frac{1}{4} - \sin^2 \theta$$

$$\text{LHS} = \cos \theta \left(\frac{1}{4} - 1 + \cos^2 \theta \right) = \cos \theta \left(\cos^2 \theta - \frac{3}{4} \right)$$

$$= \frac{1}{4} (4 \cos^3 \theta - 3 \cos \theta) = \frac{1}{4} \cos 3\theta = \text{RHS}$$

$$\text{Put } \theta = 10^\circ: \cos 10^\circ \cos 50^\circ \cos 70^\circ = \frac{1}{4} \cos 30^\circ = \frac{1}{4} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$$

$$\text{Ans: } \frac{\sqrt{3}}{8}$$

19. In $\triangle ABC$, prove that $\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C$. [5]

$$\text{Given } A + B + C = 180^\circ, \text{ so } \sin(A + B) = \sin C \text{ and } \cos(A + B) = -\cos C$$

$$\text{LHS} = 2 \sin(A + B) \cos(A - B) - 2 \sin C \cos C = 2 \sin C [\cos(A - B) - \cos C]$$

$$= 2 \sin C [\cos(A - B) + \cos(A + B)] = 2 \sin C \cdot 2 \cos A \cos B = 4 \cos A \cos B \sin C$$

$$= \text{RHS}$$

$$\text{Ans: Proved: LHS} = 4 \cos A \cos B \sin C$$

20. Find the period of $\sin \frac{x}{3} + \cos \frac{x}{4}$. [2]

Period of $\sin \frac{x}{3} = \frac{2\pi}{1/3} = 6\pi$, period of $\cos \frac{x}{4} = \frac{2\pi}{1/4} = 8\pi$

Period = L.C.M $\{6\pi, 8\pi\} = 24\pi$

Ans: 24π
